Flavour Physics: Quark Masses, Weak Couplings and CP violation in The Standard Model and Beyond

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UTfit
Extended Twisted Mass Collaboration
PRACE
INFN
Istituto Nazionale di Fisica Nucleare

electromagnetic **neutral currents** **charged currents**

$$\mathcal{L}_{int} = -eA^{\mu}J_{\mu}^{em} - \frac{g_W}{2\cos\theta_W}Z^{\mu}J_{\mu}^{Z} - \frac{g_W}{2\sqrt{2}}[W^{\mu}(J^{W})_{\mu}^{\dagger} + h.c.]$$

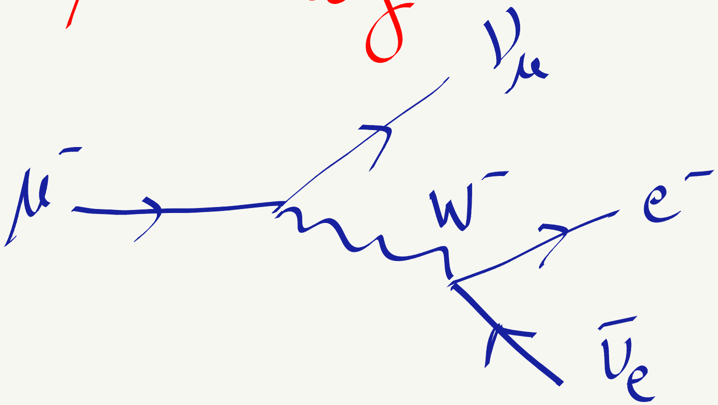
$$J_{\mu}^{Z} = 2J_{\mu}^{3} - 2\sin^2\theta_W J_{\mu}^{em}$$

$$W_{\mu}(J_W^{\dagger})^{\mu} + h.c. = W_{\mu}^{-}\,\bar{u}\gamma^{\mu}(1-\gamma_5)d + W_{\mu}^{+}\,\bar{d}\gamma^{\mu}(1-\gamma_5)u$$

+ leptons (muon decay)

[e13]

μ-decay



$$H_{eff} = -\frac{G_F}{\sqrt{2}}\, \bar{\nu}_\mu \gamma^\mu (1-\gamma_5)\mu \times \bar{e}\gamma_\mu(1-\gamma_5)\nu_e$$

$$\frac{d\Gamma}{dx} = \frac{G_F^2}{96\pi^3}\sqrt{x^2-4\rho^2}\; x(3-2x)$$

$x = 2E_e/m_\mu \qquad \rho = m_e/m_\mu$

$2\rho \le x \le 1+\rho^2$

$\rho = 0$

$$\Gamma = \frac{G_F^2 m_\mu^5}{192\pi^3}$$

Very recently, in an important theoretical development, van Ritbergen and Stuart completed the evaluation of $\mathcal{O}(\alpha^2)$ corrections to τ_μ in the local V–A theory, in the limit $m_e^2/m_\mu^2 \to 0$ [7,8]. Their final answer can be expressed succinctly as

$$C(m_\mu) = \frac{\alpha(m_\mu)}{\pi}c_1 + \left(\frac{\alpha(m_\mu)}{\pi}\right)^2 c_2, \tag{1}$$

$$c_1 = \frac{1}{2}\left(\frac{25}{4} - \pi^2\right); \qquad c_2 = 6.700. \tag{2}$$

In Eq. (1), $\alpha(m_\mu)c_1/\pi$ is the one-loop result [1] and $\alpha(m_\mu)$ is a running coupling defined by

$$\alpha(m_\mu) = \frac{\alpha}{1 - \left(\frac{2\alpha}{3\pi} + \frac{\alpha^2}{2\pi^2}\right)\ln\frac{m_\mu}{m_e}}. \tag{3}$$

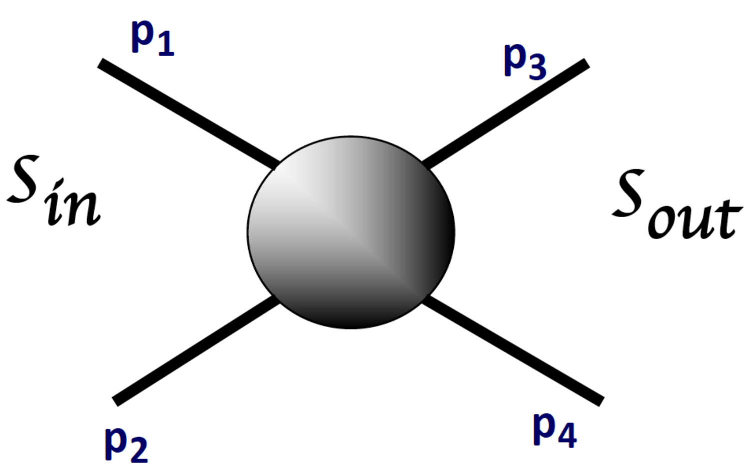
$$\Gamma = \frac{G_F^2 m_\mu^5}{192\pi^3}[1 + C(m_\mu)]f(\rho^2) + O\left(\frac{m_\mu^2}{M_W^2}\right)$$

$$f(z) = 1 - 8z - 12z^2\log z + 8z^3 - z^4$$

Consequences of a Symmetry

$[S, H] = 0 \rightarrow |E, \mathbf{p}, \mathbf{s}\rangle$

We may find states which are simultaneously eigenstates of S and of the Energy



$CP\,|K_1^0\rangle = +\,|K_1^0\rangle$

$CP\,|K_2^0\rangle = -\,|K_2^0\rangle$

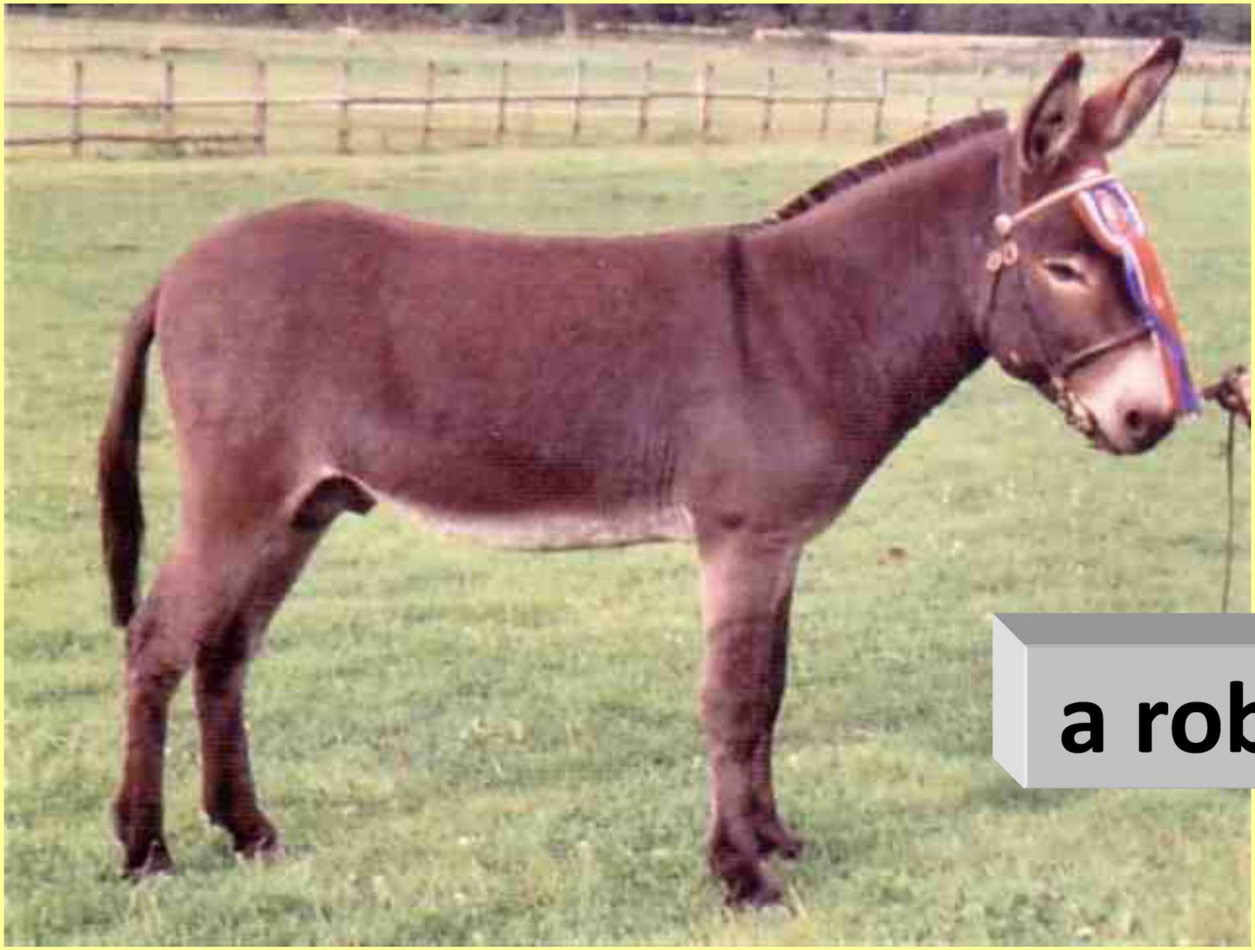
$\langle\pi\pi|K_1^0\rangle \neq 0$

$\langle\pi\pi|K_2^0\rangle = 0$

$|K_{S,L}^0\rangle = \alpha\,|K_1^0\rangle + \beta\,|K_2^0\rangle$

if CP is conserved either $\alpha=0$ or $\beta=0$

the Standard Model



a robust animal

QUARK FAMILIES

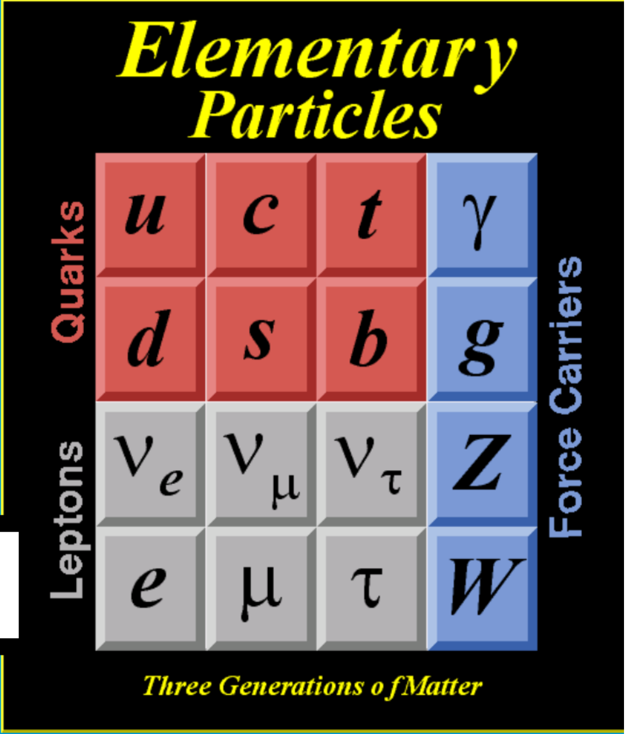
1) $q_L \equiv \begin{pmatrix} u_L \\ d_L \end{pmatrix}$ $U_R = u_R$, $D_R = d_R$

2) $q_L \equiv \begin{pmatrix} c_L \\ s_L \end{pmatrix}$ $U_R = c_R$, $D_R = s_R$

3) $q_L \equiv \begin{pmatrix} t_L \\ b_L \end{pmatrix}$ $U_R = t_R$, $D_R = b_R$

$Y=1/6$ $Y_{U,D}=Q_{U,D}=2/3,-1/3$

QUARK MASSES ARE GENERATED BY DYNAMICAL SYMMETRY BREAKING



$$H = \begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix}, \quad H^C = i\tau_2 H^*$$

$$\phi^+ \to 0 \quad \phi^0 \to \frac{V}{\sqrt{2}}$$

Charge +2/3

$$\mathcal{L}^{yukawa} \equiv \sum_{i,k=1,N} [\, Y_{i,k} (q^i_L H^C)\, U^k_R + X_{i,k} (q^i_L H)\, D^k_R + h.c. \,]$$

Charge -1/3

$$\sum_{i,k=1,N} [\, m^u_{i,k} (\bar{u}^i_L u^k_R) + m^d_{i,k} (\bar{d}^i_L d^k_R) + h.c. \,]$$

$$\mathcal{L}_{M_u} = m_u^{ij}\,\bar{u}_{iL}\,u_{jR} + (m_u^{ij})^\dagger\,\bar{u}_{iR}\,u_{jL}$$

$1+\gamma_5$ (under L); $(m_u^{ij})^\dagger \to m_u^{ji*}$; $1-\gamma_5$ (under R)

$$\to (m_u^{ij} + m_u^{ji*})\,\bar{u}^i u^j + (m_u^{ij} - m_u^{ji*})\,\bar{u}^i\gamma_5 u^j$$

P $\quad (m_u^{ij} + m_u^{ji*})\,\bar{u}^i u^j - (m_u^{ij} - m_u^{ji*})\,\bar{u}^i\gamma_5 u^j$

C $\quad (m_u^{ij} + m_u^{ji*})\,\bar{u}^j u^i + (m_u^{ij} - m_u^{ji*})\,\bar{u}^j\gamma_5 u^i$

$$= (m_u^{ji} + m_u^{ij*})\,\bar{u}^i u^j + (m_u^{ji} - m_u^{ij*})\,\bar{u}^i\gamma_5 u^j$$

CP $\quad \bar{u}\,[m_u^T + m_u^*]\,u - \bar{u}\,[m_u^T - m_u^*]\,\gamma_5 u$

$$m_u^T + m_u^* = m_u + m_u^{T*}$$
$$-m_u^T + m_u^* = m_u - m_u^{T*}$$

$$\boxed{m_u = m_u^*}$$

everything that is not forbidden is allowed

$$\sum_{i,k=1,N} [\ m^u_{i,k} (\overline{u^i_L}\, u^k_R) \quad + m^d_{i,k} (\overline{d^i_L}\, d^k_R) + h.c.\ ]$$

It is easy to show the a necessary and sufficient condition for CP invariance is

$$m^{u,d}_{i,k} = \text{real}$$

1) there is no compelling symmetry for $m^{u,d}_{i,k}$ to be real
2) in field theory, all that may happen will happen [see below]
3) symmetries and accidental symmetries e.g. separate conservation of lepton and baryon numbers (it follows from gauge symmetry and renormalizability)

DIAGONALIZATION MASS MATRIX

non singular $\underline{M}$ $\det M \neq 0$

$$\boxed{M_D = U_L^\dagger M U_R}$$

with real and positive eigenvalues

$$M = HU = U'H'$$

$A = M^\dagger M$ $B = MM^\dagger$

$A^\dagger = A$ $B^\dagger = B$

$A_D = U_R^\dagger A U_R$ $B_D = U_L^\dagger B U_L$

unitary matrices U_R, U_L

$$\det(M^\dagger M - \lambda) = 0$$

$$= \det(M^\dagger - \lambda M^{-1}) \det M = \det(MM^\dagger - \lambda)$$

$$(M^\dagger)' M' = U_R^\dagger M^\dagger U_L U_L^\dagger M U_R = U_R^\dagger M^\dagger M U_R = A_D$$

$$M'(M')^\dagger = U_L^\dagger M M^\dagger U_L = B_D$$

$$M = \frac{M + M^\dagger}{2} + i\,\frac{M - M^\dagger}{2i} \leftarrow R + i\mathcal{Y}$$

hermitian matrices

$$[M'_+ M^{\dagger\prime}, M'_- M^{\dagger\prime}] = -[M', M^{\dagger\prime}]$$

$$+ [M^{\dagger\prime}, M'] = 2[M^{\dagger\prime}, M'] =$$

$$2\,(M^{\dagger\prime} M' - M' M^{\dagger\prime}) = B_D - A_D = 0$$

$$M'_D = \tilde{U}^\dagger M \tilde{U} = \underbrace{\tilde{U}^\dagger U_L^\dagger}_{U_L^\dagger} M \underbrace{U_R \tilde{U}}_{U_R}$$

$$M'_D = U_L^\dagger M U_R =$$

$$M_D \quad \begin{pmatrix} m_1 e^{i\delta_1} & & & \\ & m_2 e^{i\delta_2} & & \\ & & \cdots & \\ & & & m_N e^{i\delta_N} \end{pmatrix} =$$

$$\begin{pmatrix} e^{i\delta_1} & & & \\ & e^{i\delta_2} & & \\ & & \cdots & \\ & & & e^{i\delta_N} \end{pmatrix} \begin{pmatrix} m_1 & & & \\ & m_2 & & \\ & & \cdots & \\ & & & m_N \end{pmatrix}$$

$$M = HU \qquad H = U_1 M_D U_1^\dagger$$

$$H = U_1 U_L^\dagger M U_R U_1^\dagger$$

$$U_L U_1^\dagger H U_1 U_R^\dagger = M$$

$U_1 = U_L$ $\qquad M = H(U_L U_R^\dagger) = HU$

$U_1 = U_R$ $\qquad M = (U_L U_R^\dagger) H = U' H$

Diagonalization of the Mass Matrix

The mass matrix (M) is not even Hermitean;
Up to singular cases, however, it can always be diagonalized
by 2 unitary transformations

$$u^i_L \ \square\ U^{ik}_L\, u^k_L \qquad\qquad u^i_R \ \square\ U^{ik}_R\, u^k_R$$

$$M' = U^{\dagger}_L\, M\, U_R \qquad\qquad (M')^{\dagger} = U^{\dagger}_R\, (M)^{\dagger}\, U_L$$

$$L^{\mathrm{mass}} \equiv m_{up}\,(\bar{u}_L\, u_R + \bar{u}_R\, u_L) + m_{ch}(\bar{c}_L\, c_R + \bar{c}_R\, c_L) + m_{top}(\bar{t}_L\, t_R + \bar{t}_R\, t_L)$$

$$L_{CC}^{\mathrm{weak\ int}} = \frac{g_W}{2\sqrt{2}}\,(J^-_\mu W^+_\mu + J^+_\mu W^-_\mu)$$

The Cabibbo-Kobayashi-Maskawa Matrix

$$J^{+}_{\mu} W^{-}_{\mu} = (\bar{u}\,\gamma_{\mu}(1 - \gamma_{5})\,d + \ldots)\, W^{-}_{\mu} =$$

$$\bar{u}_{L}\,\gamma_{\mu}\,d_{L}\,W^{-}_{\mu} \rightarrow \bar{u}_{L}\,(\mathbf{U_{L}^{u}})^{\dagger}{}_{L}\,\gamma_{\mu}\,(\mathbf{U_{L}^{d}})\,d_{L}\,W^{-}_{\mu} =$$

$$\bar{u}_{L}\,\mathbf{V^{CKM}}\,\gamma_{\mu}\,d_{L}\,W^{-}_{\mu}$$

where $\mathbf{V^{CKM} = (U_{L}^{u})^{\dagger}\,(U_{L}^{d})}$ is a unitary matrix

With N(N-1)/2 Euler angles and [N^2 - N(N-1)/2]

phases; not all these phases are physical

neutral currents remain diagonal in flavor (no FCNC at tree level)

$u_L \rightarrow e^{i\phi_u}\, u_L \qquad d_L \rightarrow e^{i\phi_d}\, d_L$ etc.

Thus finally we have N(N-1)/2 angles and (N-1)(N-2)/2 phases

The Cabibbo-Kobayashi-Maskawa Matrix

Up to singular cases, the mass matrix can always be diagonalized by 2 unitary transformations

$$u^i_L \to U^{ik}_L u^k_L \qquad u^i_R \to U^{ik}_R u^k_R$$

$$M' = U^\dagger_L M U_R \qquad (M')^\dagger = U^\dagger_R (M)^\dagger U_L$$

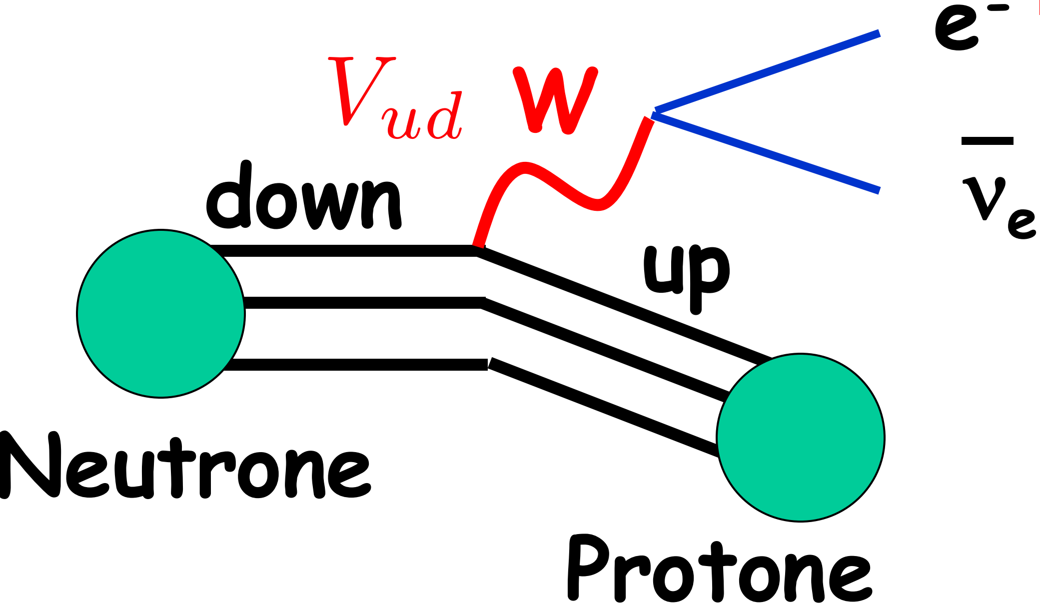
$$L^{\mathbf{mass}} \equiv m_{\mathbf{up}} (\bar{u}_{\mathbf{L}} u_{\mathbf{R}} + \bar{u}_{\mathbf{R}} u_{\mathbf{L}}) + m_{\mathbf{ch}} (\bar{c}_{\mathbf{L}} c_{\mathbf{R}} + \bar{c}_{\mathbf{R}} c_{\mathbf{L}}) + m_{\mathbf{top}} (\bar{t}_{\mathbf{L}} t_{\mathbf{R}} + \bar{t}_{\mathbf{R}} t_{\mathbf{L}})$$

$$L_{CC}^{weak\,int} = \frac{g_W}{\sqrt{2}} \left(J^-_\mu W^+_\mu + h.c. \right) \rightarrow \frac{g_W}{\sqrt{2}} \left(\bar{u}_L \mathbf{V}^{CKM} \gamma_\mu d_L W^+_\mu + \ldots \right)$$

Weak Interactions

β-Decays

$$n \rightarrow p + e^{-} + \bar{\nu}_e$$



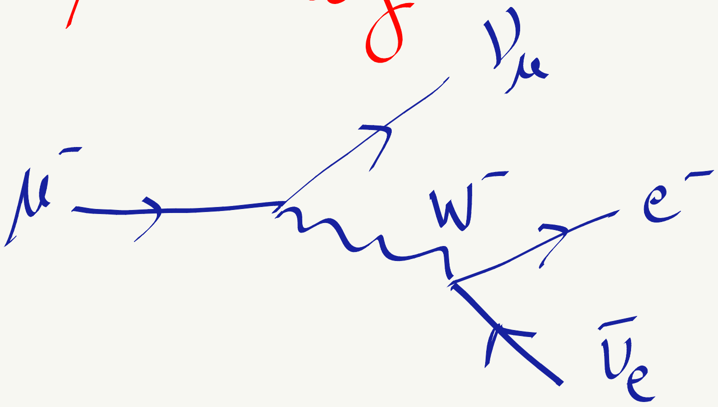
$$^{60}_{27}\mathrm{Co} \rightarrow {}^{60}_{28}\mathrm{Ni} + e^{-} + \bar{\nu}_e$$

$$\mathcal{H}_{eff} = -\frac{G_F}{\sqrt{2}} [\bar{u}\gamma_\mu(1-\gamma_5)d][\bar{e}\gamma^\mu(1-\gamma_5)\nu_e]$$

$$i\frac{-g_{\mu\nu} + q_\mu q_\nu / M_W^2}{q^2 - M_W^2 + i\varepsilon} \sim i\frac{g_{\mu\nu}}{M_W^2} \qquad q^2 << M_W^2 \qquad \frac{G_F}{\sqrt{2}} = \frac{g_W^2}{8M_W^2}$$

μ-decay

[e13]



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$$f(z) = 1 - 8z - 12z^2\log z + 8z^3 - z^4$$

N(N-1)/2 angles and (N-1)(N-2) /2 phases

N=3 3 angles + 1 phase KM

the phase generates complex couplings i.e. <u>*CP* violation;</u>

6 masses +3 angles +1 phase = 10 parameters

V_{ud}	V_{us}	V_{ub}
V_{cd}	V_{cs}	V_{cb}
V_{tb}	V_{ts}	V_{tb}

$$N^2 - \frac{N(N-1)}{2} - (2N-1) = \frac{(N-1)(N-2)}{2}$$

$$= \begin{bmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta_{13}} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta_{13}} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta_{13}} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta_{13}} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta_{13}} & c_{23}c_{13} \end{bmatrix}$$

NO Flavour Changing Neutral Currents (FCNC) at Tree Level
(FCNC processes are good candidates for observing NEW PHYSICS)

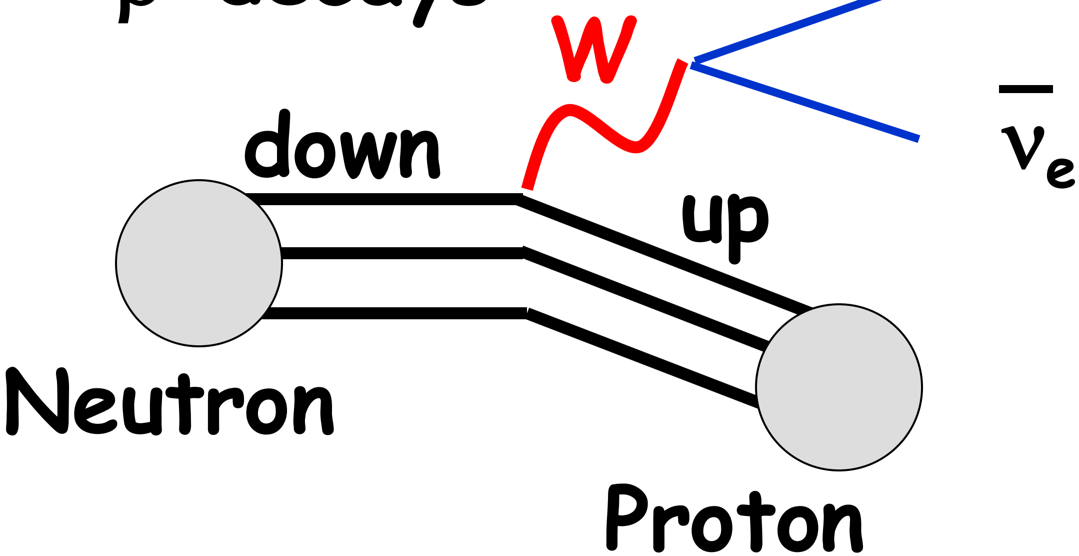
CP Violation is natural with three quark generations (Kobayashi-Maskawa)

With three generations all CP phenomena are related to the same unique parameter (δ)

V_{ud}	V_{us}	V_{ub}
V_{cd}	V_{cs}	V_{cb}
V_{td}	V_{ts}	V_{tb}

Quark masses & Generation Mixing

β-decays



$$|V_{ud}| = 0.9735(8)$$
$$|V_{us}| = 0.2196(23)$$
$$|V_{cd}| = 0.224(16)$$
$$|V_{cs}| = 0.970(9)(70)$$
$$|V_{cb}| = 0.0406(8)$$
$$|V_{ub}| = 0.00409(25)$$
$$|V_{tb}| = 0.99(29)$$
$$|V_{td}| \quad (0.999)$$

updated values next slide

CKM December 2022

$$\begin{pmatrix} 0.97431(19) & 0.22517(81) & 0.003715(93)\,e^{-i(65.1(1.3))^\circ} \\ -0.22503(83)\,e^{+i(0.0351(1))^\circ} & 0.97345(20)\,e^{-i(0.00187(5))^\circ} & 0.0420(5) \\ 0.00859(11)\,e^{-i(22.4(7))^\circ} & -0.04128(46)\,e^{+i(1.05(3))^\circ} & 0.999111(20) \end{pmatrix}$$

Quark Masses from Lattice QCD

$$m_\nu \leq 1\, eV$$

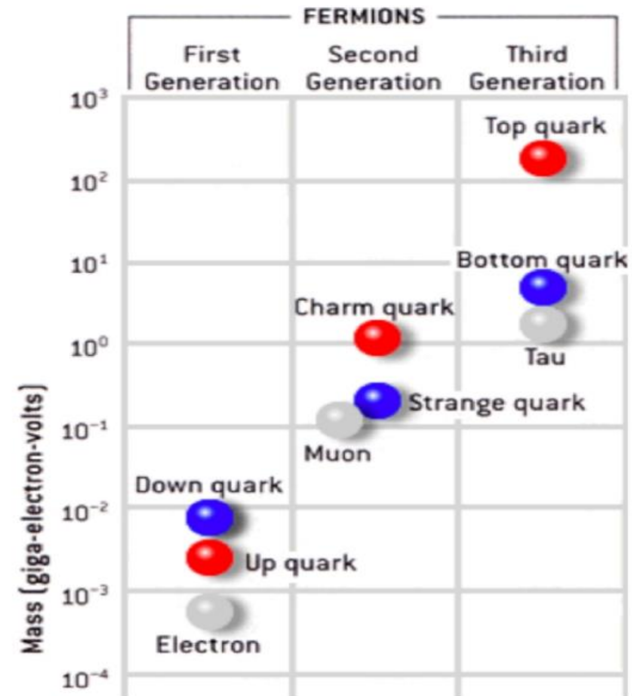


Illustration from a G. Isidori talk

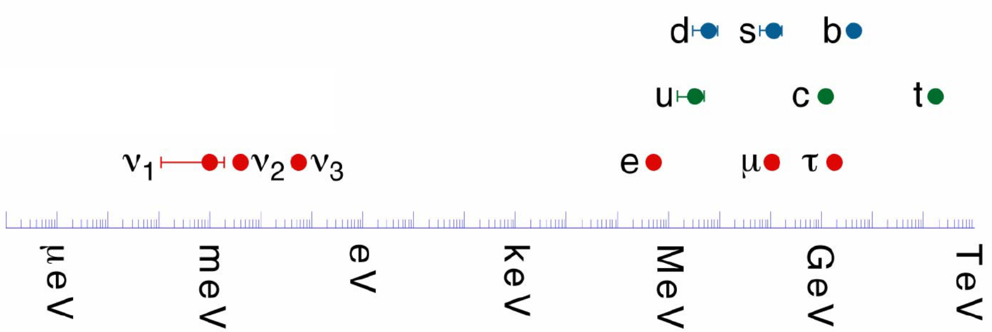
Input	Lattice/Exp
$m_u^{\overline{\rm MS}}(2\,{\rm GeV})$	2.20(9) MeV
$m_d^{\overline{\rm MS}}(2\,{\rm GeV})$	4.69(2) MeV
$m_s^{\overline{\rm MS}}(2\,{\rm GeV})$	93.14(58) MeV
$m_c^{\overline{\rm MS}}(3\,{\rm GeV})$	993(4) MeV
$m_c^{\overline{\rm MS}}(m_c^{\overline{\rm MS}})$	1277(5) MeV
$m_b^{\overline{\rm MS}}(m_b^{\overline{\rm MS}})$	4196(19) MeV
$m_t^{\overline{\rm MS}}(m_t^{\overline{\rm MS}})$ (GeV) to be updated	163.44(43)

Table 3 *Full lattice inputs. The values of the different quantities have been obtained by taking the weighted average of the* $N_f = 2+1$ *and* $N_f = 2+1+1$ *FLAG numbers.*

Questions

how to extend the SM in order to accommodate neutrino masses?

why neutrino masses are so small, compared with the charged fermion masses?



why lepton mixing angles are so different from those of the quark sector?

$$U_{PMNS} = \begin{pmatrix} \frac{2}{\sqrt{6}} & \frac{1}{\sqrt{3}} & 0 \\ -\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & -\frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} \end{pmatrix} + \text{corrections}$$

$$V_{CKM} \approx \begin{pmatrix} 1 & O(\lambda) & O(\lambda^4 \div \lambda^3) \\ O(\lambda) & 1 & O(\lambda^2) \\ O(\lambda^4 \div \lambda^3) & O(\lambda^2) & 1 \end{pmatrix}$$

$\lambda \approx 0.22$

Textures

There is a clear correlation between mixings and masses

$m_u \sim 3$ MeV $m_c \sim 1200$ MeV $m_t \sim 170$ GeV

$m_d \sim 7$ MeV $m_s \sim 110$ MeV $m_b \sim 4.3$ GeV

Orizontal $\mathcal{U}(2)$: ψ_L $\psi_L{}^c$

$$\mathcal{L}_{higgs} = \frac{Y\,H}{M}\left[\,(\psi_L{}^a)(\psi_L{}^b)^c\, S^{a\,b} + (\psi_L{}^a)(\psi_L{}^b)^c\, A^{a\,b}\,\right]$$

Symmetric tensor

Antisymmetric tensor

$$M^d = M \begin{pmatrix} 0 & -\sqrt{x} \\ \sqrt{x} & 1+x \end{pmatrix}$$

$$\mathrm{Sin}\,\theta_c \sim \sqrt{m_d / m_s}$$

R.Gatto '70

$$\mathrm{diag}(M) = M\,(x\ ,\ 1)$$

$$x = m_d\ /\ m_s$$

$$V_1 = \begin{pmatrix} 1 \\ \sqrt{x} \end{pmatrix} \qquad \lambda_1 = M\,x$$

$$V_2 = \begin{pmatrix} -\sqrt{x} \\ 1 \end{pmatrix} \qquad \lambda_2 = M$$

Masses & Mixings (including th
CP phases)
are related !

$$\begin{bmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta_{13}} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta_{13}} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta_{13}} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta_{13}} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta_{13}} & c_{23}c_{13} \end{bmatrix}$$

$c_{ij} = \mathrm{Cos}\ \theta_{ij}$ $s_{ij} = \mathrm{Sin}\ \theta_{ij}$ $c_{ij} \geq 0$ $s_{ij} \geq 0$

$0 \leq \delta \leq 2\pi$ $|s_{12}| \sim \mathrm{Sin}\ \theta_c$

for small angles $|s_{ij}| \sim |V_{ij}|$

The Wolfenstein Parametrization

$1 - 1/2\, l^{2}$	l	$A\, l^{3}(r - i\, h)$
$- l$	$1 - 1/2\, l^{2}$	$A\, l^{2}$
$A\, l^{3}$ ´ $(1- r - i\, h)$	$-A\, l^{2}$	1

V_{ub}

$+ O(\lambda^4)$

V_{td}

$\sin\theta_{12} = \lambda$

$\sin\theta_{23} = A\,\lambda^2$

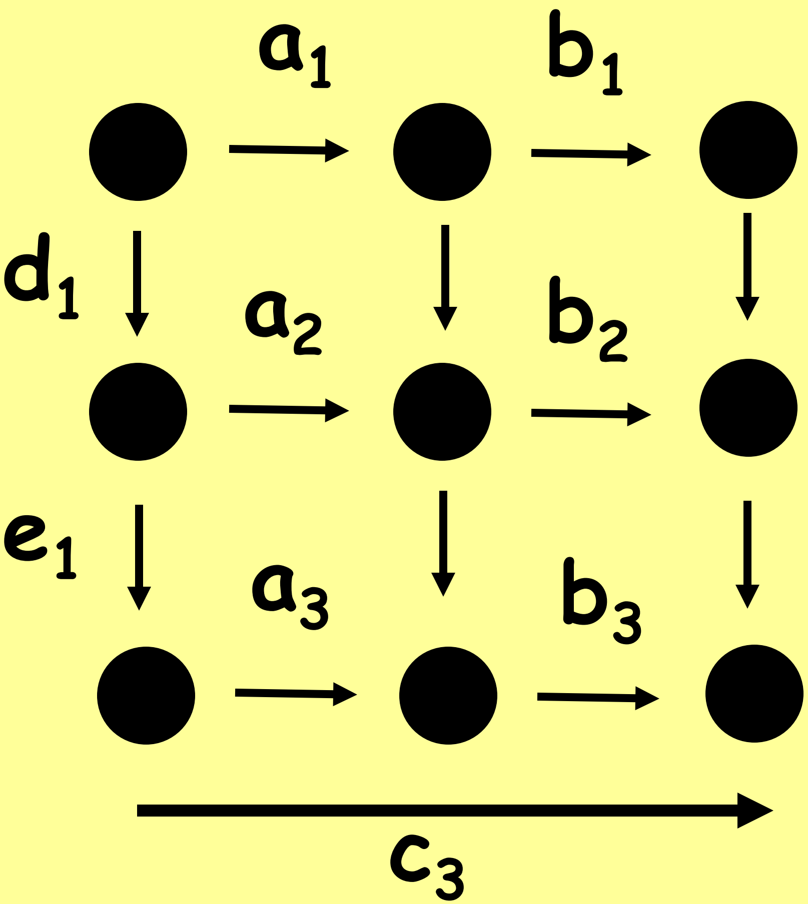
$\sin\theta_{13} = A\,\lambda^3(\rho - i\,\eta)$

$\lambda \sim 0.2 \quad A \sim 0.8$

$\eta \sim 0.2 \quad \rho \sim 0.3$

The Bjorken-Jarlskog Unitarity Triangle

$|V_{ij}|$ is invariant under phase rotations



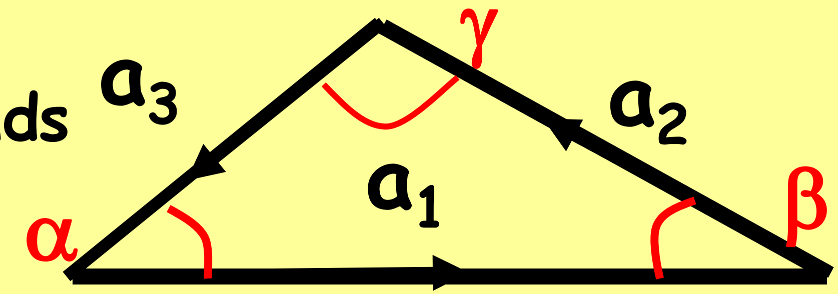
$a_1 = V_{11}V_{12}^* = V_{ud}V_{us}^*$

$a_2 = V_{21}V_{22}^*$ $a_3 = V_{31}V_{32}^*$

$a_1 + a_2 + a_3 = 0$

$(b_1 + b_2 + b_3 = 0$ etc.)

Only the orientation depends on the phase convention



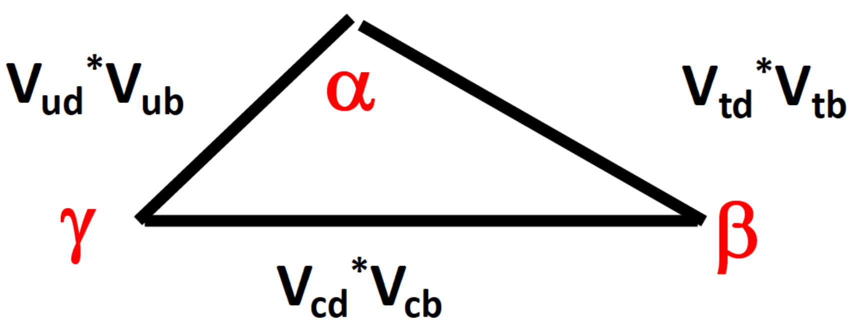
Physical quantities correspond to invariants under phase reparametrization i.e. $|a_1|, |a_2|, \ldots, |e_3|$ and the area of the Unitary Triangles

$$J = \mathrm{Im}\,(a_1 a_2^{*}) = |a_1\, a_2| \sin\beta$$

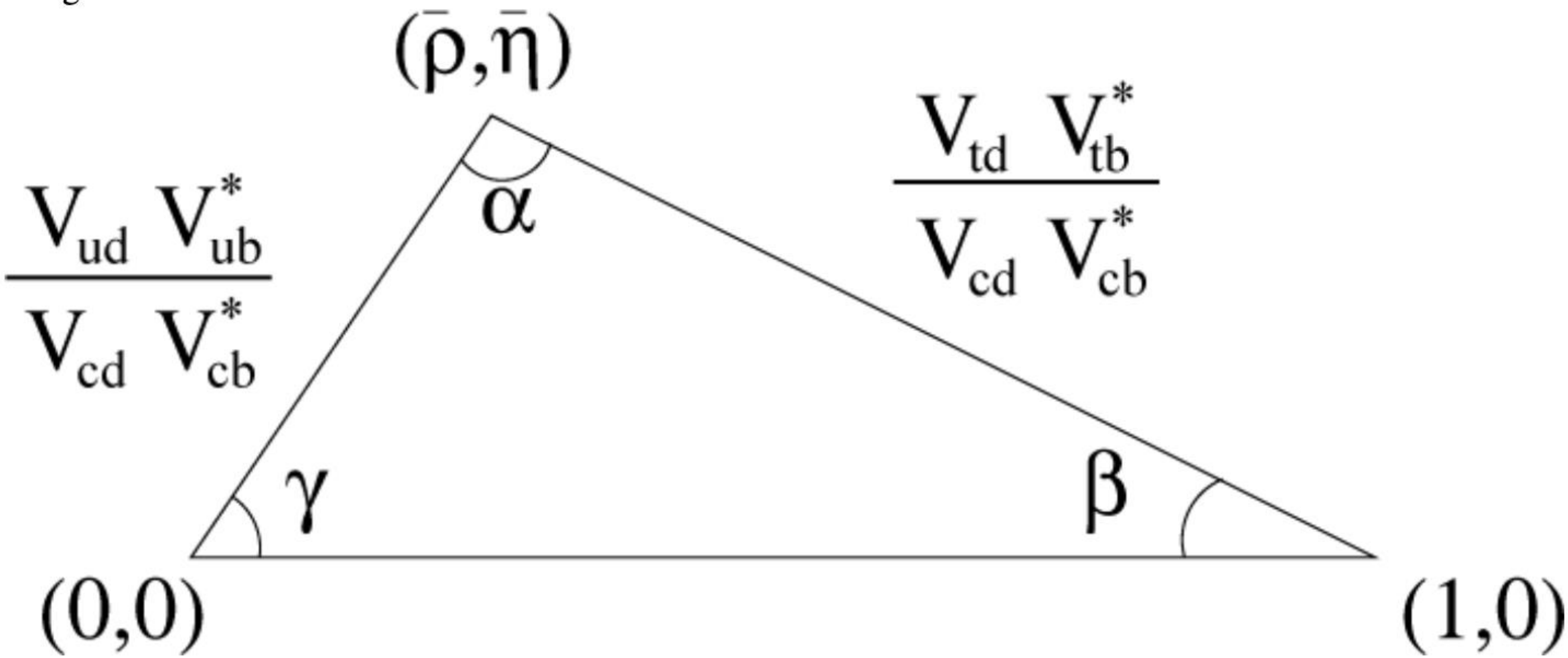
a precise knowledge of the moduli (angles) would fix J

$$\not{CP} \propto J$$

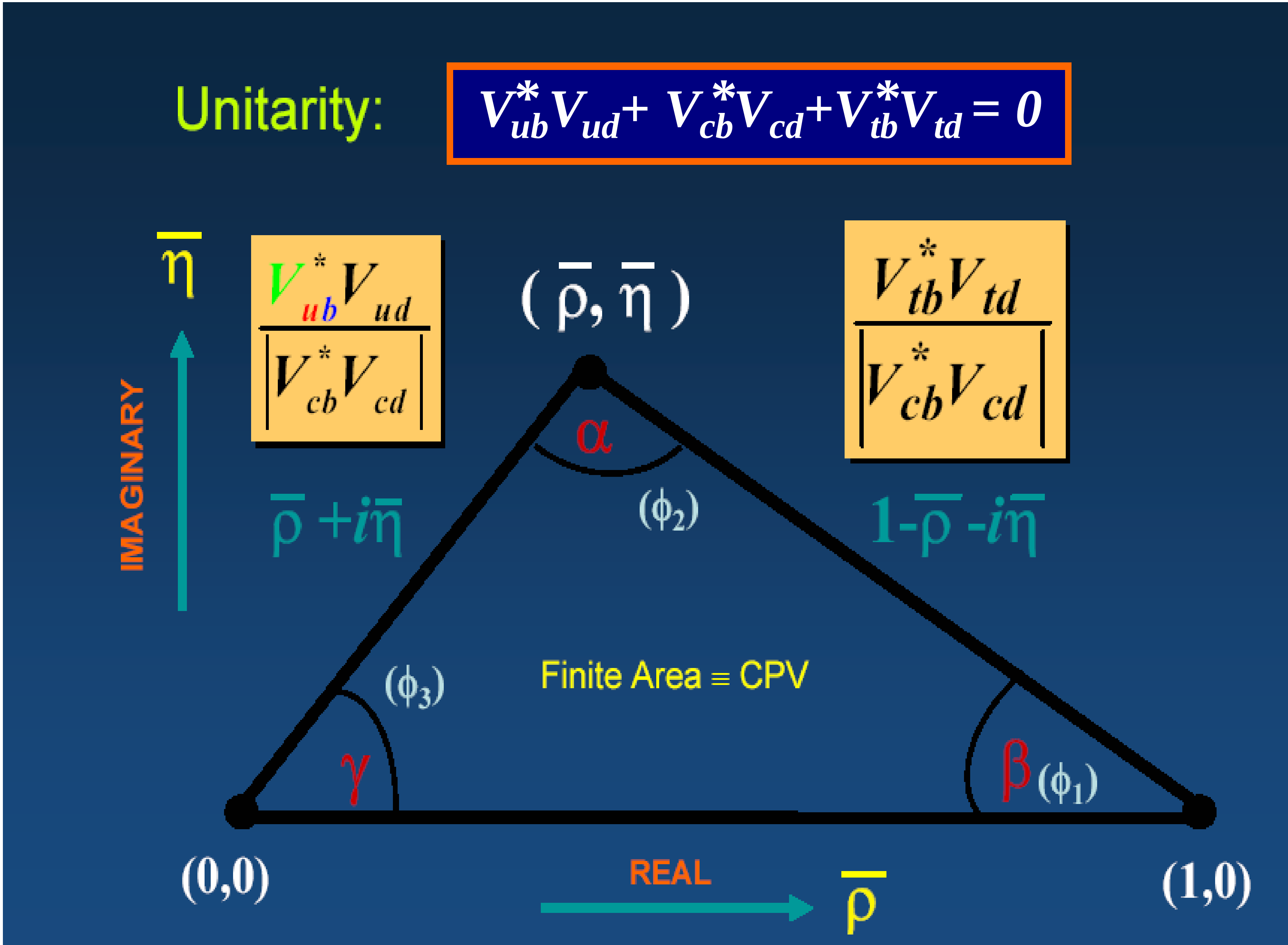
$$V_{ud}^{*}V_{ub} + V_{cd}^{*}V_{cb} + V_{td}^{*}V_{tb} = 0$$



$$\gamma = \delta_{CKM}$$



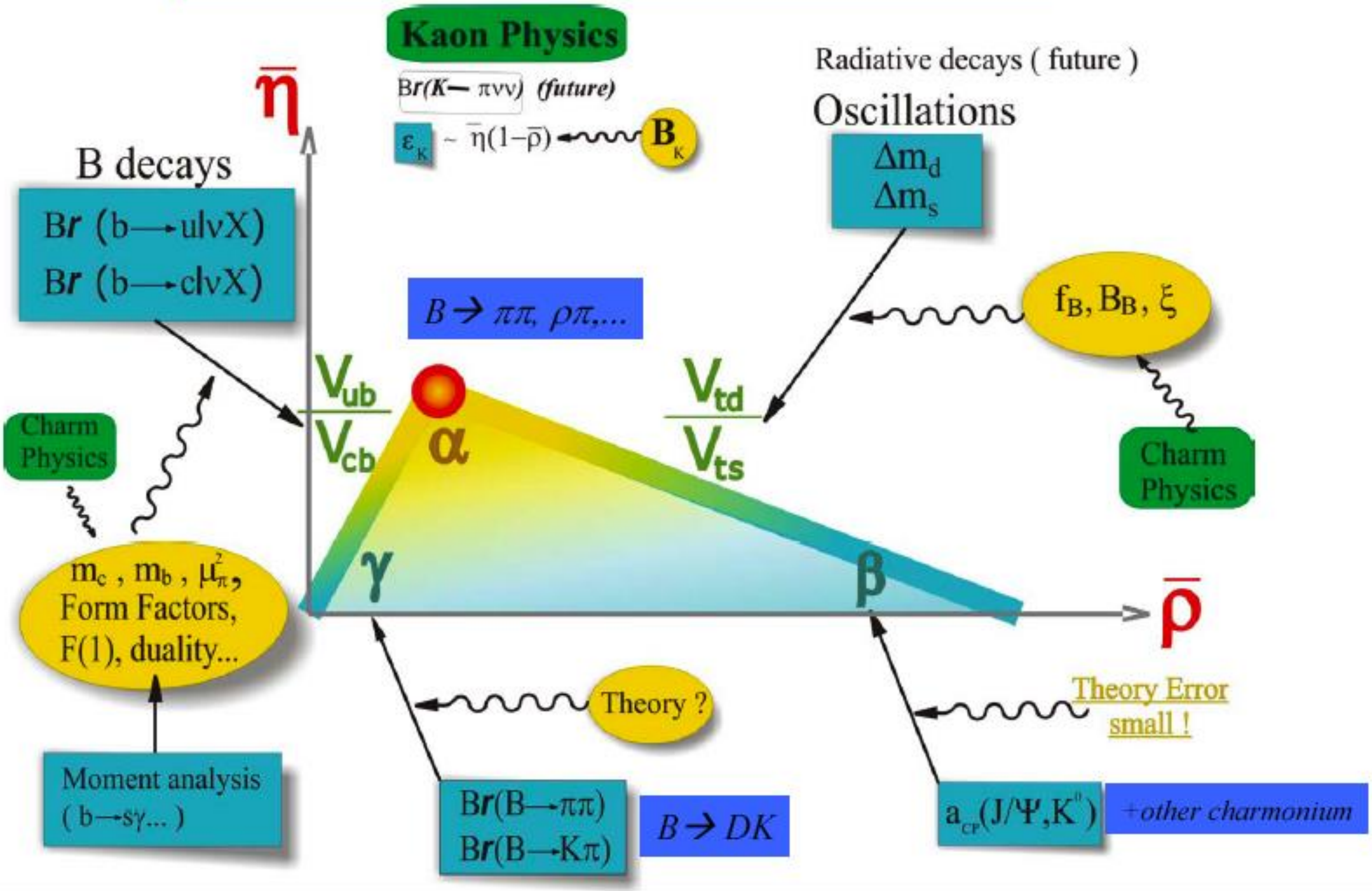
The Standard Triangle of the Standard Model


Unitarity:
$V_{ub}^{*}V_{ud}+V_{cb}^{*}V_{cd}+V_{tb}^{*}V_{td}=0$
$\frac{V_{ub}^{*}V_{ud}}{|V_{cb}^{*}V_{cd}|}$
$(\bar{\rho},\bar{\eta})$
$\frac{V_{tb}^{*}V_{td}}{|V_{cb}^{*}V_{cd}|}$
$\bar{\eta}$
IMAGINARY
α
(ϕ_2)
$\bar{\rho}+i\bar{\eta}$
$1-\bar{\rho}-i\bar{\eta}$
(ϕ_3)
Finite Area ≡ CPV
γ
$\beta\ (\phi_1)$
(0,0)
REAL
$\bar{\rho}$
(1,0)

Visualization of the unitarity of the CKM matrix

Unitarity Triangle in the (ρ–η) plane

From
A. Stocchi
ICHEP 2002



STRONG CP VIOLATION

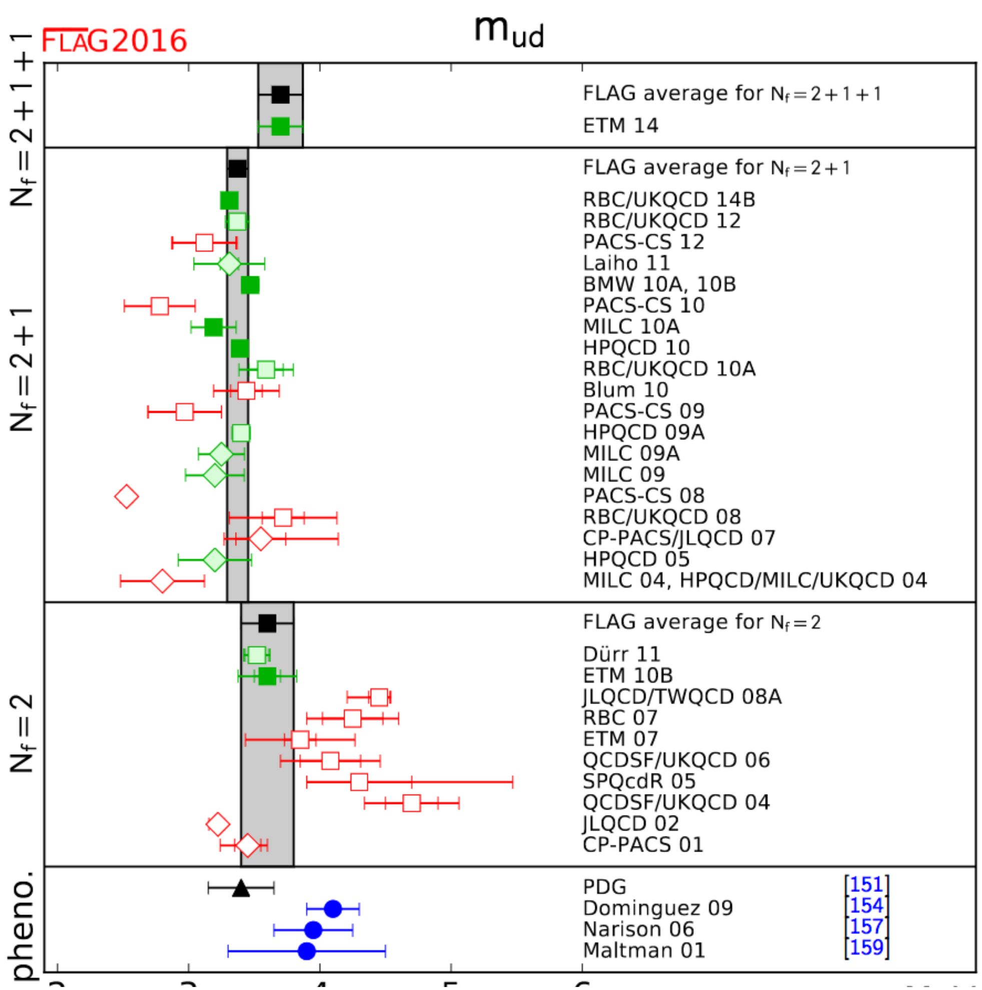
$$L_\theta = \theta \, \tilde{G}^{\mu\nu a} G^a_{\mu\nu} \qquad \tilde{G}^a_{\mu\nu} = \varepsilon_{\mu\nu\rho\sigma} G^a_{\rho\sigma}$$

$$L_\theta \sim \theta \, \vec{E}^a \cdot \vec{B}^a$$

This term violates CP and gives a contribution to the electric dipole moment of the neutron

$$e_n < 3 \; 10^{-26} \text{ e cm}$$

$\theta < 10^{-10}$ which is quite unnatural !!

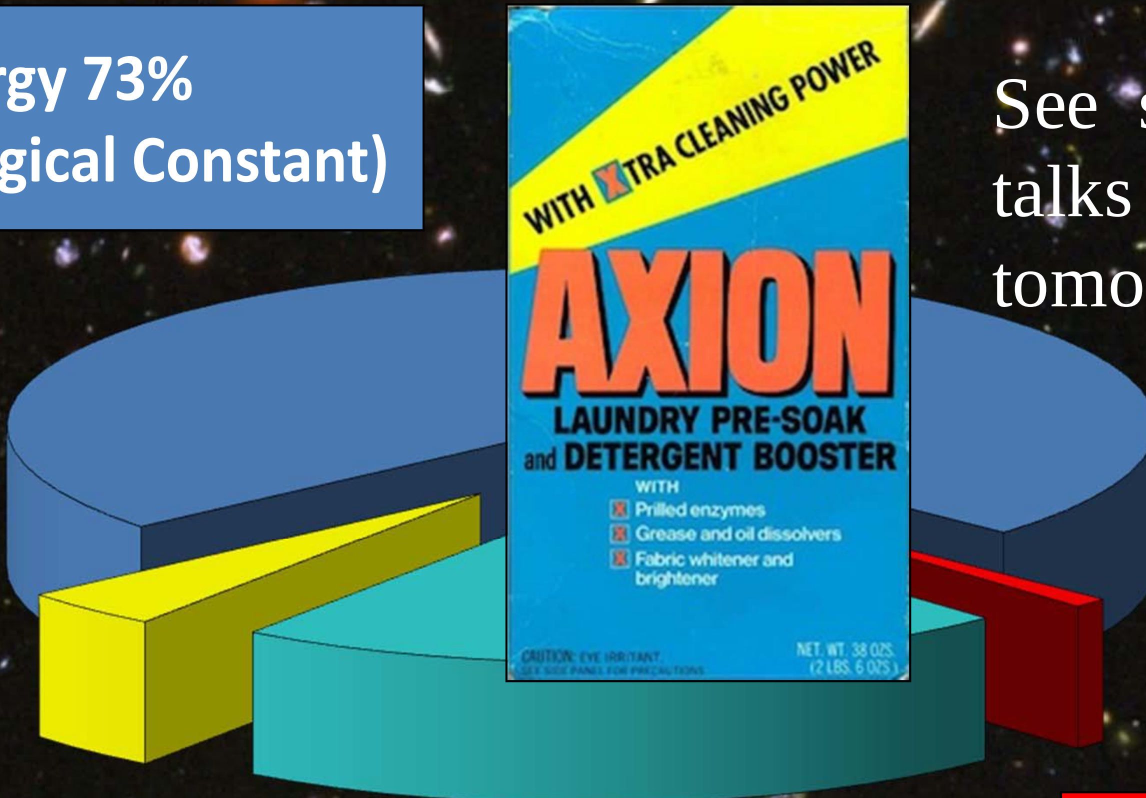


N_f	m_u	m_d	m_u/m_d	R	Q
2+1+1	2.36(24)	5.03(26)	0.470(56)	35.6(5.1)	22.2 (1.6)
2+1	2.16(9)(7)	4.68(14)(7)	0.46(2)(2)	35.0(1.9)(1.8)	22.5(6)(6)
2	2.40(23)	4.80(23)	0.50(4)	40.7(3.7)(2.2)	24.3(1.4)(0.6)

Raffelt

See several talks on axions tomorrow

Dark Energy 73%
(Cosmological Constant)

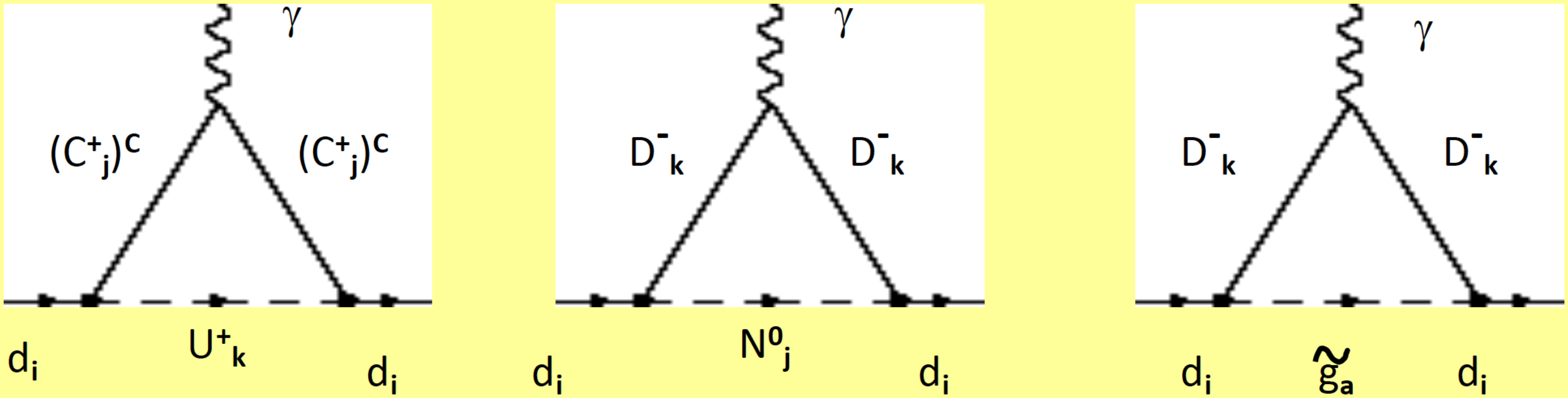


Ordinary Matter 4%
(of this only about
10% luminous)

Dark Matter
23%

Neutrinos
0.1- 2%

Neutron electric dipole moment in SuperSymmetry



$$\mathcal{L}^{\Delta F=0} = -i/2\, C_e\, \overline{\psi}\sigma_{\mu\nu}\gamma_5\psi\, F^{\mu\nu}$$
$$-i/2\, C_c\, \psi\sigma_{\mu\nu}\gamma_5\, t^a\overline{\psi}\, G^{\mu\nu a}$$
$$-1/6\, C_g\, f_{abc}\, G^a{}_{\mu\rho}\, G^{b\rho}{}_{\nu}\, G^c{}_{\lambda\sigma}\, \varepsilon^{\mu\nu\lambda\sigma}$$

$C_{e,C,g}$ can be computed perturbatively

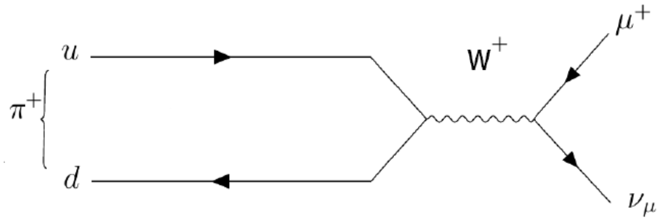
Classification of the processes in the SM

Leptonic Decays

the prototype of these decays is given by

$$\pi^+ \longrightarrow \mu^+ + \nu_\mu \tag{35}$$

which at the fundamental level is given by



Other possible leptonic decays are given by

$$K^+ \longrightarrow \mu^+ + \nu_\mu$$
$$D^+ \longrightarrow \mu^+ + \nu_\mu$$
$$B^+ \longrightarrow \tau^+ + \nu_\tau$$
$$\pi^+ \longrightarrow e^+ + \nu_e$$

the latter process is suppressed by chirality

Semi-leptonic Decays

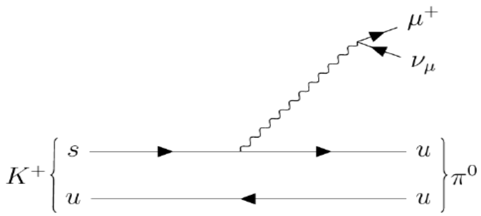
these are the better sources to measure the absolute values of the CKM matrix elements |V ij|

$$n \longrightarrow p + e^- + \overline{\nu_e}$$

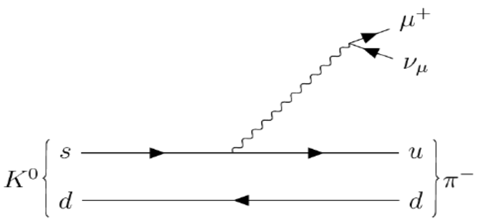
other possible semi-leptonic decays are the following

$$\pi^+ \longrightarrow \pi^0 + e^+ + \nu_e$$

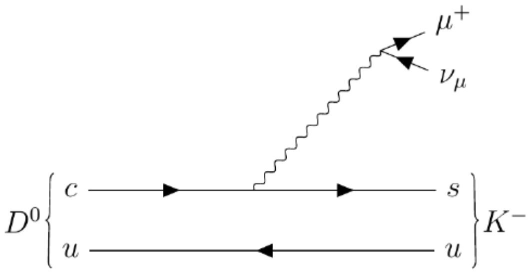
$$K^+ \longrightarrow \pi^0 + \mu^+ + \nu_\mu$$



$$K^0 \longrightarrow \pi^- + \mu^+ + \nu_\mu$$



$$D^0 \longrightarrow K^+ + \mu^+ + \nu_\mu$$

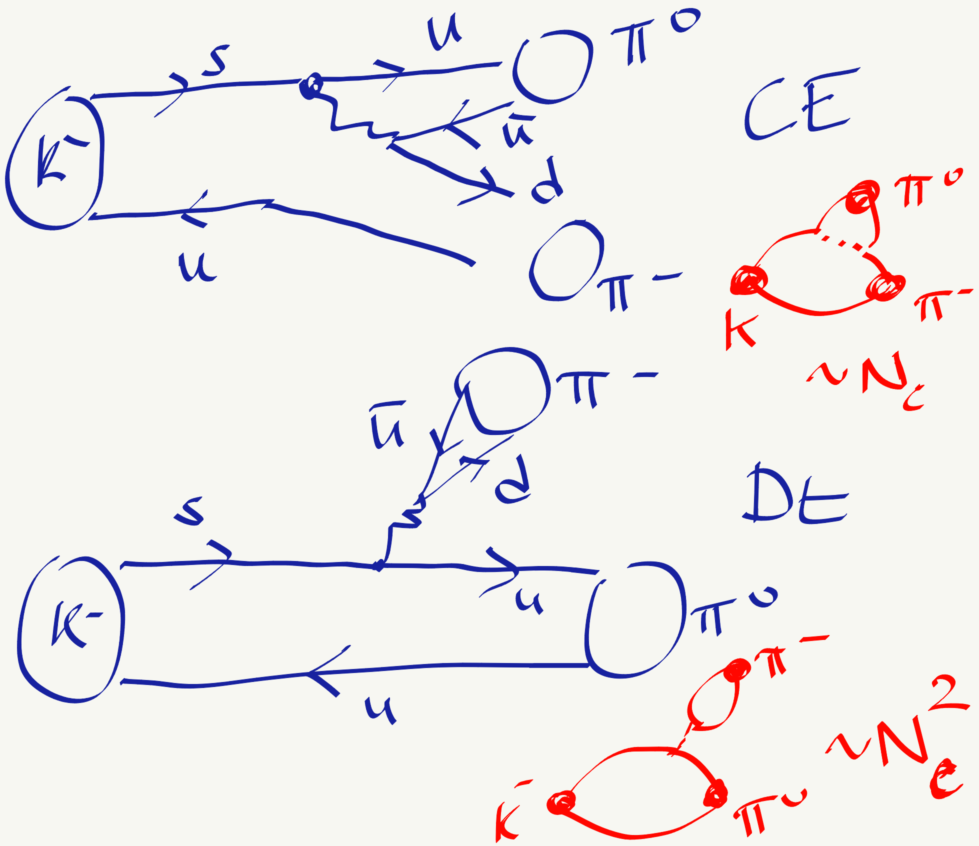


Non-leptonic Decays

Penguins contractions and all that

$$K^- \to \pi^- \pi^0$$

$$H_W = -\frac{G_F}{\sqrt{2}} V_{us} V_{ud}^* \, \bar{u}\gamma^\mu(1-\gamma_5)s \, \bar{d}\gamma_\mu(1-\gamma_5)u$$

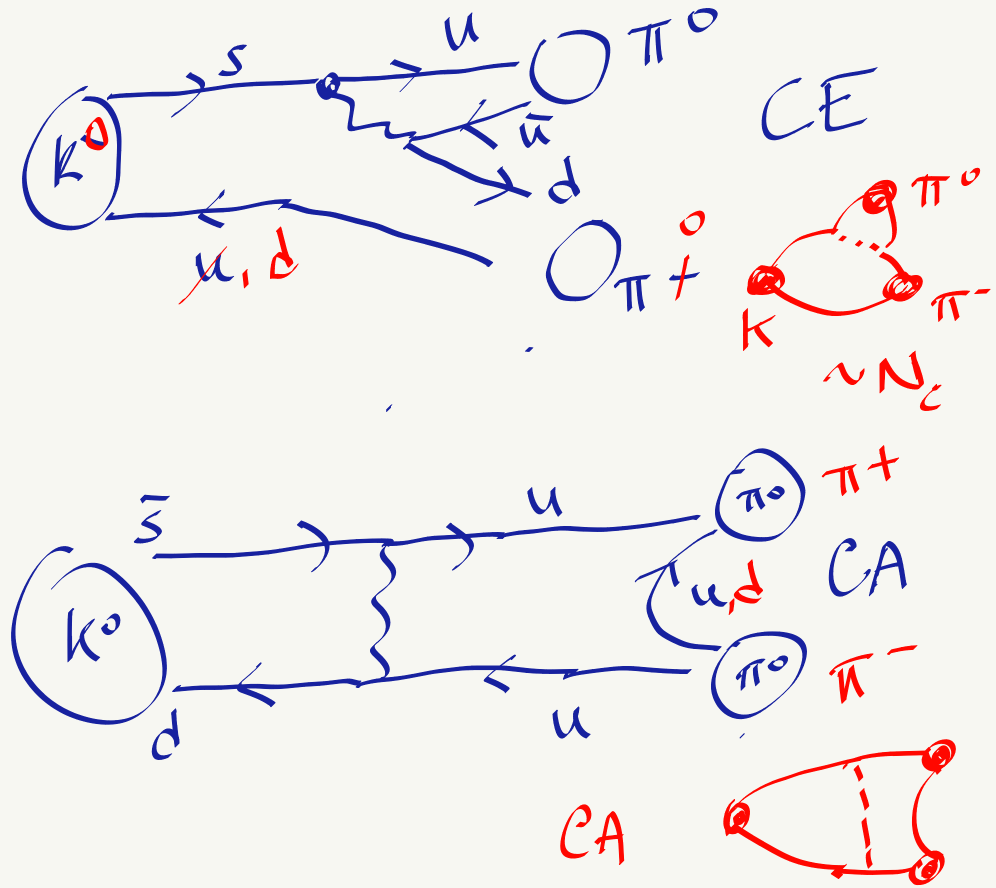


Non-leptonic Decays

Penguins contractions and all that

$$K^{+/0} \to \pi^{+/0} \pi^0$$

$$H_W = -\frac{G_F}{\sqrt{2}} V_{us} V_{ud}^* \, \bar{u} \gamma^\mu (1-\gamma_5) s \, \bar{d} \gamma_\mu (1-\gamma_5) u$$

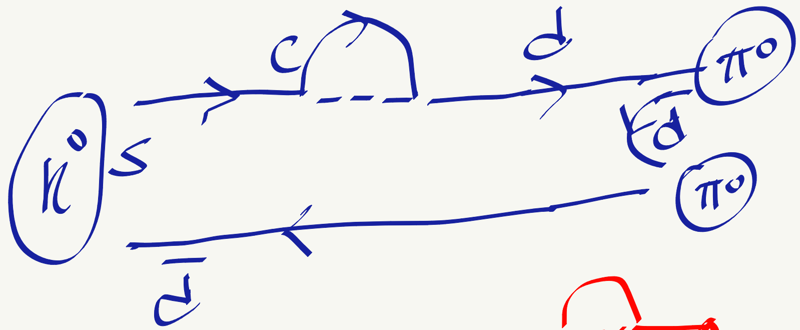


Non-leptonic Decays

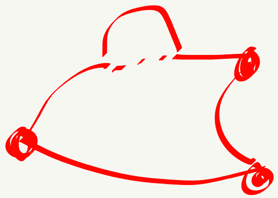
Penguins contractions and all that

Penguins diagrams

$$H = -\frac{G_F}{\sqrt{2}} V_{cs} V_{cd}^{*} \, \bar{c}\gamma_\mu(1-\gamma_5)s \, \bar{d}\gamma^\mu(1-\gamma_5)c$$



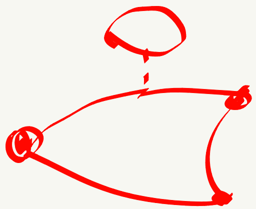
CP



CP

other ops

$$\sim V_{cs} V_{cd}^{*} \quad \bar{d}\gamma_\mu(1-\gamma_5)s \, \bar{c}\gamma^\mu(1-\gamma_5)c$$



DP

All Topologies

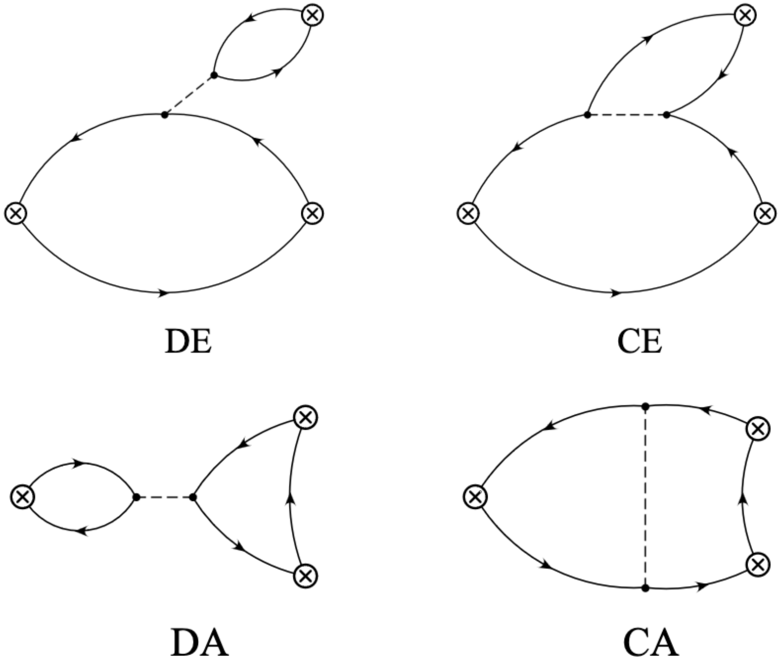
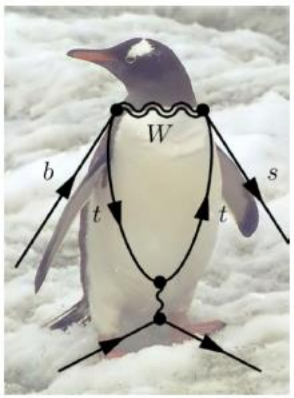


Figure 1: *Non-penguin diagrams. The dashed line represents the four-fermion operator.*

All Topologies



D. Kovalskyi's seminar @ CERN (26/7/22)

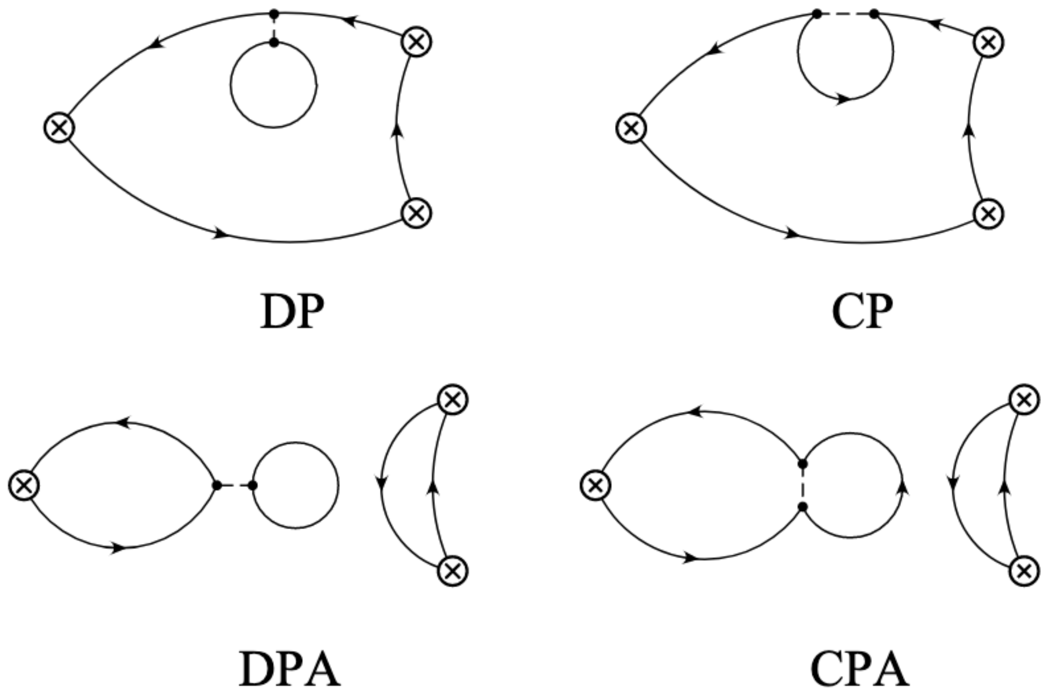
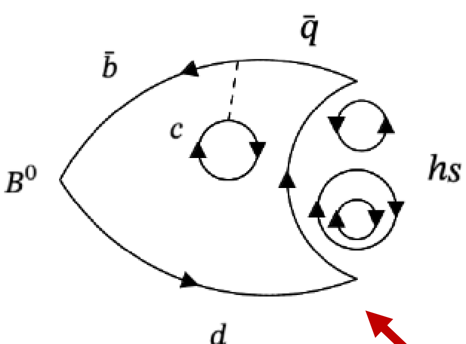


Figure 2: *Penguin diagrams.*



for heavy mesons many particles in the final state

Rare Penguin Radiative Decays

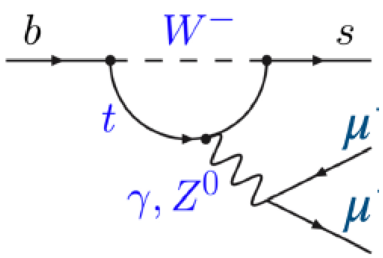
The main issue in this Group Meeting will be represented by b → s quark transitions: the best example is offered by the neutral-current semileptonic B → K(*) l+ l- transitions!

Many interesting properties:

1. **Loop-level processes (FCNCs are forbidden at tree level in the Standard Model)**
2. **CKM-suppressed decays, where**

$$J^{\mu}_{\text{charged}} = \bar{u}^i_L V^{ij}_{CKM} \gamma^\mu d^j_L + \bar{\nu}^i_L \gamma^\mu \ell^i_L$$

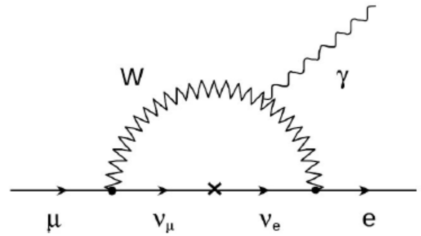
Rare transitions!



L. Vittorio (LAPTh & CNRS, Annecy)

$$B^+ \to K^{(*)+}\gamma \qquad B^+ \to K^{(*)+}\mu^+\mu^-$$

since different neutrinos have a mass and they can mix, $\mu \to e\gamma$ is a possible decay which satisfies all the symmetry constraints



$$\mathcal{B}(\mu \to e\gamma) \sim \alpha \frac{m_\nu^4}{m_W^4} \sim 10^{-52}$$

note that the photon is emitted by the W boson, analogy radiative B decays

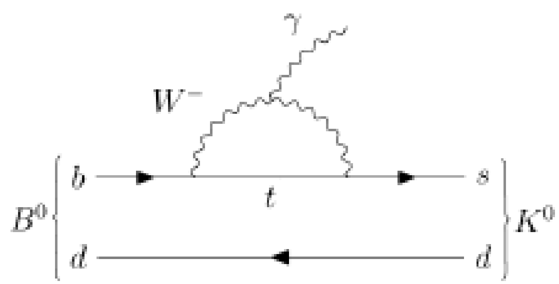


Figura 4: quark process

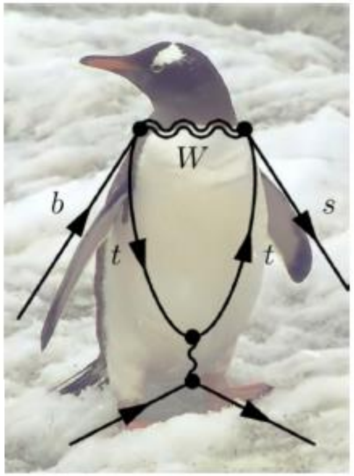
Radiative Penguins

PENGUINS AND BOXES

Pure leptonic Bs decays

$$Br(B_s \to l^+ l^-) = \tau(B_s)\frac{G_F^2}{\pi}\left(\frac{\alpha}{4\pi \sin^2\Theta_W}\right)^2 \boxed{F_{B_s}^2} m_l^2 m_{B_s}\sqrt{1-4\frac{m_l^2}{m_{B_s}^2}}|V_{tb}^* V_{ts}|^2 Y^2(x_t)$$

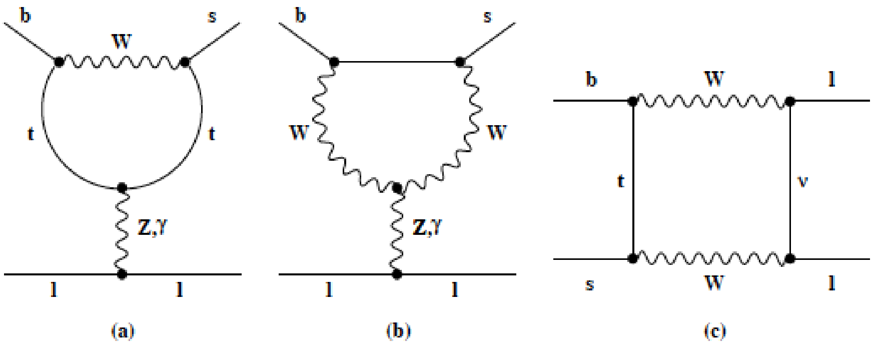
G. Buchalla and A.J. Buras, Nucl. Phys. B 400 (1993) 225.



valskyi's seminar @ CERN (26/7/22)

Many interesting properties:

1. Helicity suppressed
2. Non-perturbative hadronic contributions enter via Bs decay constant



Lowest order diagrams
QCD corrections at NLO or NNLO

BOXES

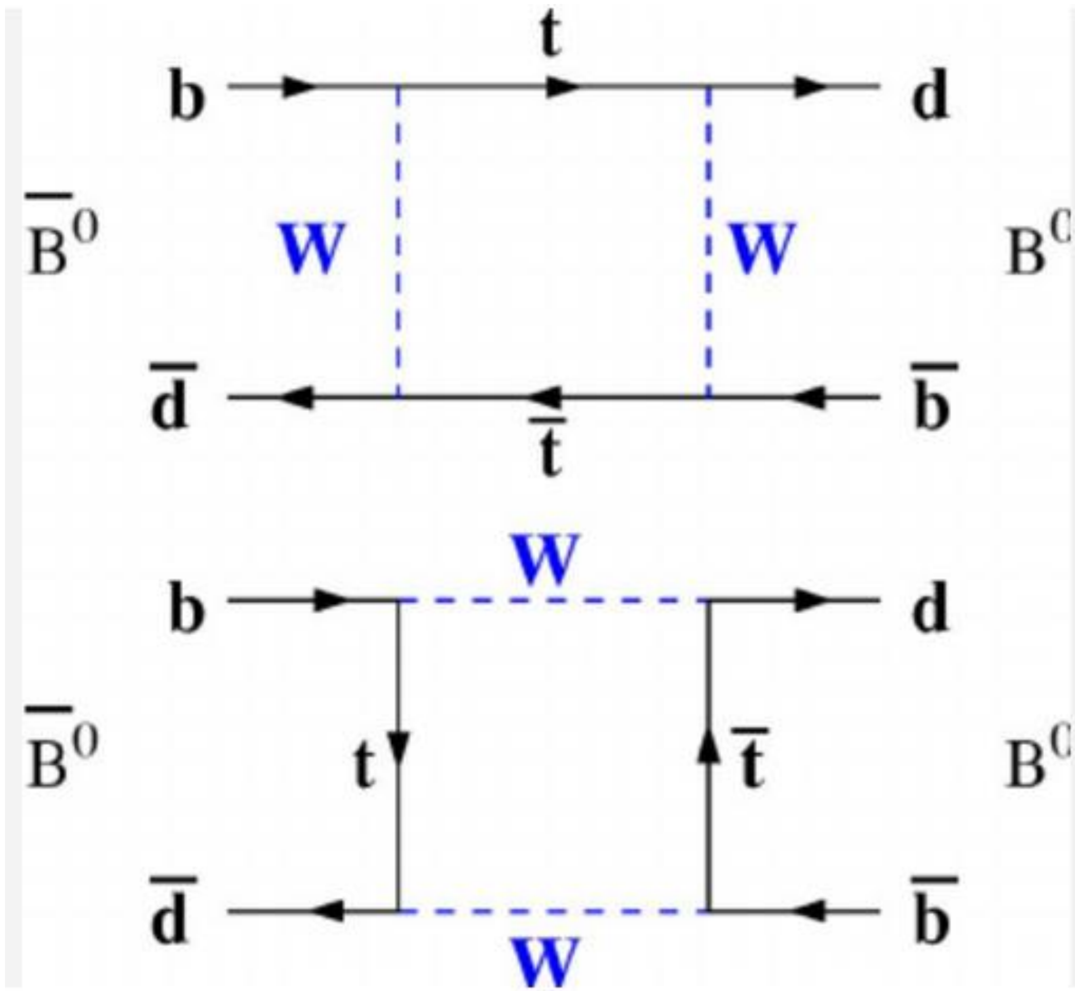
Mixing of Neutral Mesons

$$K^0 \leftrightarrow \bar{K}^0$$

$$D^0 \leftrightarrow \bar{D}^0$$

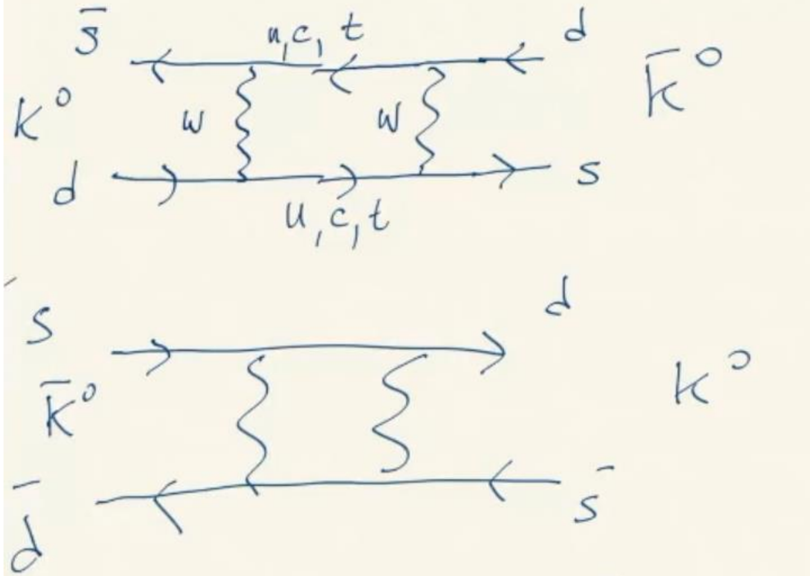
$$B^0 \leftrightarrow \bar{B}^0$$

in the case of kaons
also charm and up quarks
contribute
for D and K meson mixing
there are important long
distance contributions



$$\mathcal{H}_{\rm eff}^{\Delta B=2} = \frac{G_{\rm F}^2}{16\pi^2} M_W^2 \left(V_{tb}^* V_{tq}\right)^2 \eta_B S_0(x_t) \times$$
$$\times \left[\alpha_s^{(5)}(\mu_b)\right]^{-6/23} \left[1 + \frac{\alpha_s^{(5)}(\mu_b)}{4\pi} J_5\right] Q^q(\Delta B = 2) + h.c.$$

QCD corrections



$|K^0\rangle$; $|\bar{K}^0\rangle$

$$H = \begin{pmatrix} M & 0 \\ 0 & M \end{pmatrix} \quad CPT$$

$$H = \begin{pmatrix} M & H_{12} \\ H_{21} & M \end{pmatrix}$$

if $H_{12} = H_{21}$

$$(M-\lambda)^2 - H_{12}^2 = 0$$

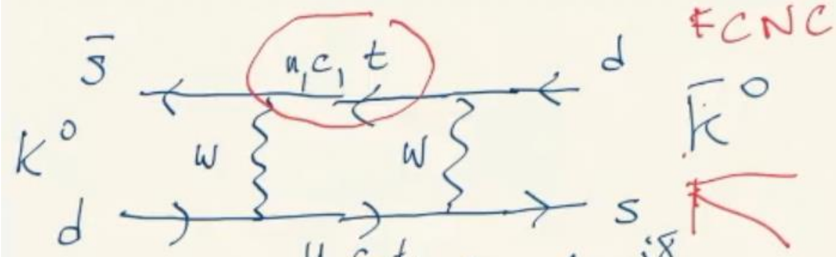
$$M_\pm = \lambda_\pm = M \pm H_{12}$$

$$|K^+\rangle = \frac{|K^0\rangle + |\bar{K}^0\rangle}{\sqrt{2}}$$

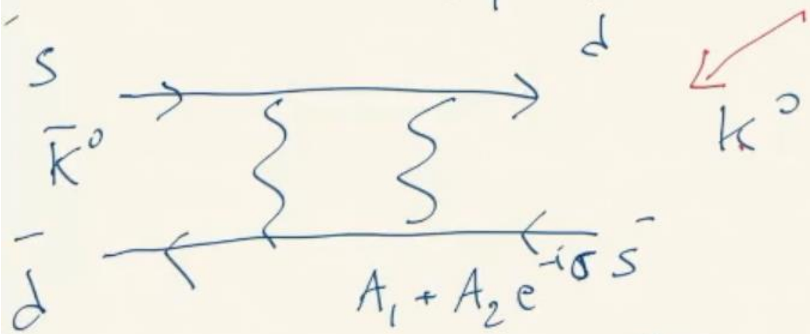
$$|K^-\rangle = \frac{|K^0\rangle - |\bar{K}^0\rangle}{\sqrt{2}}$$

$$CP|K^0\rangle = |\bar{K}^0\rangle$$

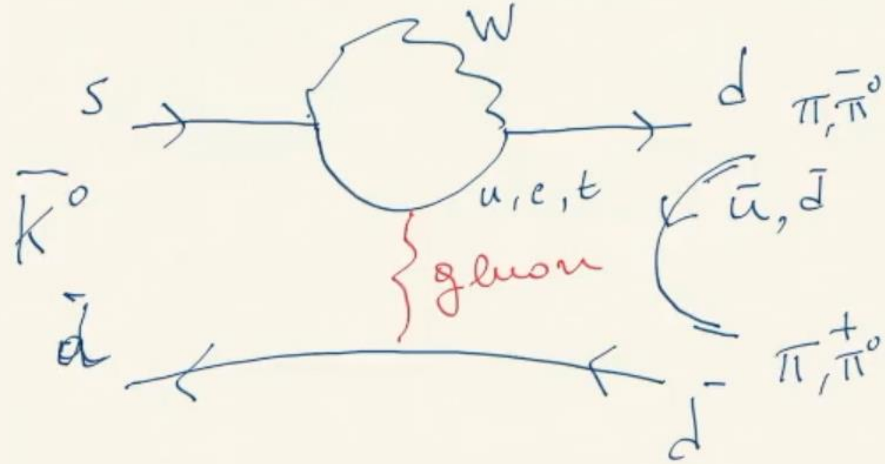
$$CP|\bar{K}^0\rangle = |K^0\rangle$$



$$A_1 + A_2 e^{i\delta}$$



$$A_1 + A_2 e^{-i\delta}$$



$$= A_u + A_c + A_t$$

$$= V_{us} V_{ud}^* I_u + V_{cs} V_{cd}^* I_c + V_{ts} V_{td}^* I_t$$

$$CP|K^+\rangle = |K^+\rangle$$

$$CP|K^-\rangle = -|K^-\rangle$$

$$CP|\pi^0\pi^0\rangle_S = (-1)^{\ell=0}\, \eta_{\pi^0}^2\, |\pi^0\pi^0\rangle$$

$$\eta_{\pi^0}^2 = (-1)^2$$

$$= |\pi^0\pi^0\rangle \qquad CP=+1$$

$$CP|\pi^+\pi^-\rangle_S = |\pi^+\pi^-\rangle_S$$

if CP is a symmetry of the SM

$$K^+ \to \pi^+\pi^-,\ \pi^0\pi^0$$

$$K^- \not\to \pi^+\pi^-,\ \pi^0\pi^0$$

$CP=-1$ $\qquad$ $CP=+1$

$$|K\rangle \sim |K^-\rangle$$

$$|K\rangle = \alpha|K^0\rangle + \beta|\bar{K}^0\rangle$$

$$= c_1|K^+\rangle + c_2|K^-\rangle$$

$|K^+\rangle \to \pi^0\pi^0,\ \pi^+\pi^-$ $\qquad$ $|K^-\rangle \leftarrow \pi^+\pi^-\pi^0,\ \pi^0\pi^0\pi^0$

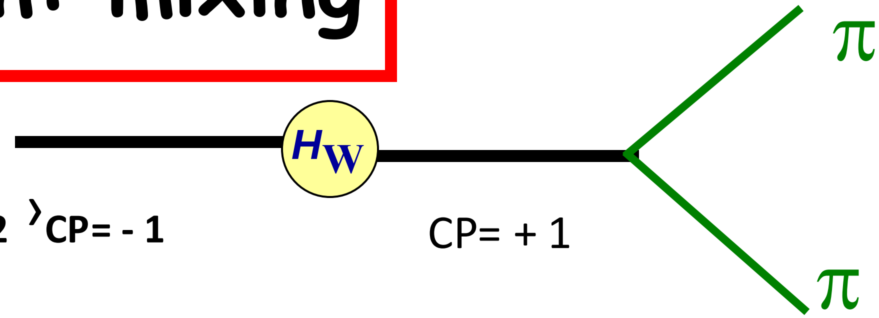
$$\tau_+ \ll \tau_-$$

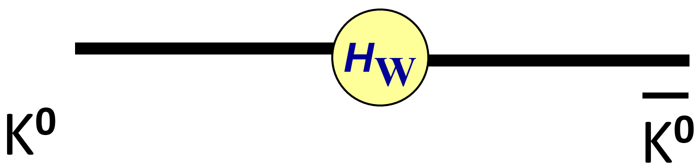
$$|K(t)\rangle = \cancel{c_1 e^{-t/\tau_+}|K_+\rangle} + c_2 e^{-t/\tau_-}|K^-\rangle$$

Indirect CP violation: mixing

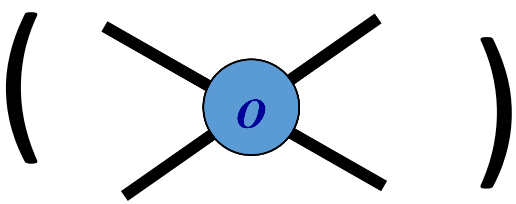
ε_K

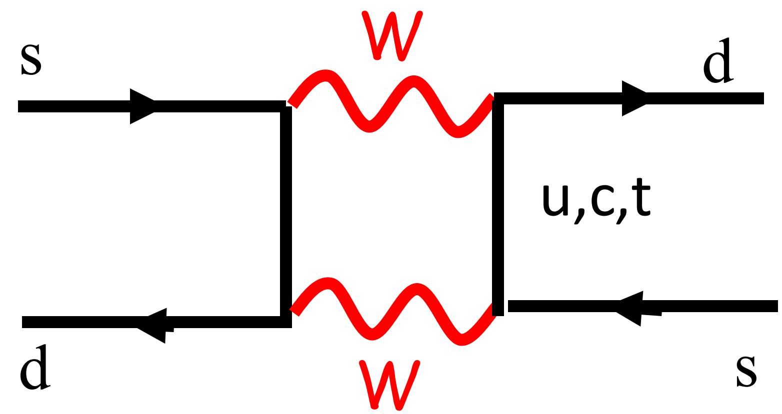
$|K_L\rangle = |K_2\rangle_{CP=-1}$





ΔS=2

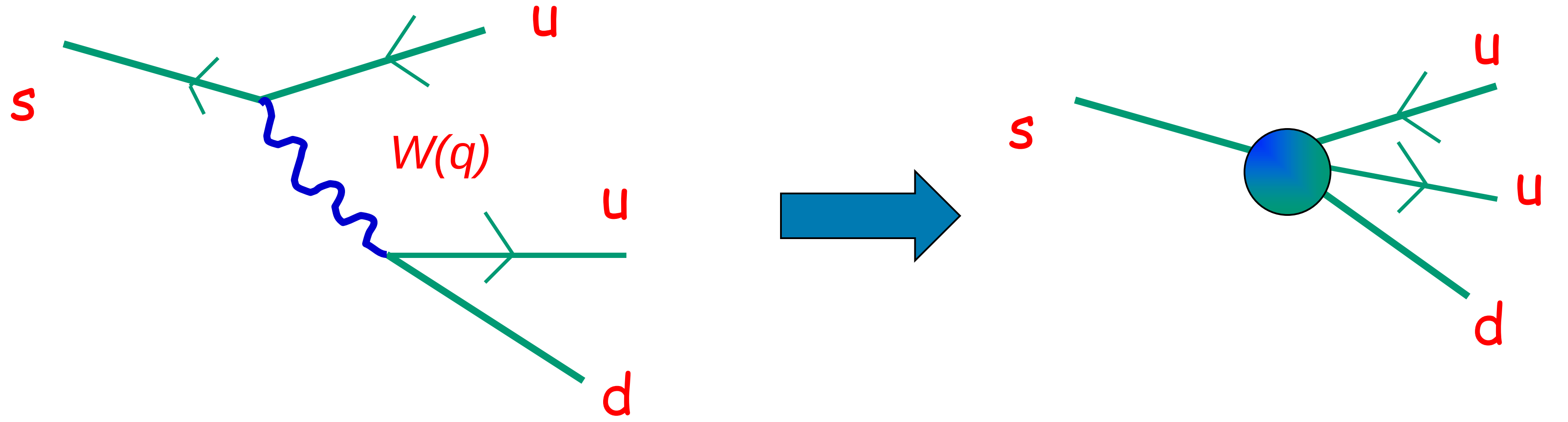




Box diagrams:
They are also responsible for B^0 - $\overline{B}^0$ mixing
$\Delta m_{d,s}$

Complex ΔS=2 effective coupling

The Effective Hamiltonian, Wilson OPE and QCD Corrections



$$q \sim m_K \ll M_W$$

$$\mathcal{H}_{eff} = -\frac{G_F}{\sqrt{2}} V_{ud} V_{us}^* \; (\bar{s}\gamma_\mu(1-\gamma_5)u)\,(\bar{u}\gamma^\mu(1-\gamma_5)d)$$

GENERAL FRAMEWORK: THE OPE

$$A_{FI}\,(2\pi^4)\,\delta^4\,(p_F - p_I) = \int d^4x\, d^4y\, D_{\mu\nu}(x, M_W)$$
$$\langle F\,|T[\,J_\mu(y+x/2)\, J^\dagger_\nu(y-x/2)]\,|\,I\rangle$$

$$\langle F\,|\,H^{\Delta S=1}\,|\,I\rangle =$$

$$G_F/\sqrt{2}\, V_{ud}\, V_{us}^* \qquad \Sigma_i\, C_i(\mu)\, \frac{\langle F\,|\,Q_i(\mu)\,|\,I\rangle}{(M_W)^{di-6}}$$

di= dimension of the operator $Q_i(\mu)$

$C_i(\mu)$ Wilson coefficient: it depends on M_W/μ and $\alpha_W(\mu)$

$Q_i(\mu)$ local operator renormalized at the scale μ

GENERAL FRAMEWORK

$$H^{\Delta S=1} = G_F/\sqrt{2}\, V_{ud}\, V_{us}^{*} \Big[(1-\tau) \Sigma_{i=1,2}\, z_i \big(Q_i - Q^c_i\big) + \tau\, \Sigma_{i=1,10} \big(z_i + y_i \big)\, Q_i \;\Big]$$

Where y_i and z_i are short distance coefficients, which are known in perturbation theory at the NLO (Buras et al. + Ciuchini et al.)

$$\tau = -V_{ts}^{*} V_{td} / V_{us}^{*} V_{ud}$$

We have to compute $A^{I=0,2}{}_i = \langle\, (\pi\, \pi)_{I=0,2}\, |\, Q_i\, |\, K \,\rangle$ with a non perturbative technique (lattice, QCD sum rules, 1/N expansion etc.)

$$A_0 = \sum_i \; C_i(\mu) \; \langle (\pi\,\pi) \,|\, Q_i(\mu) \,|\, K \rangle_{I=0} \; (1 - \Omega_{IB})$$

μ = renormalization scale
μ-dependence cancels if operator matrix elements are consistently computed

Isospin Breaking

$$\mathcal{A}_2 = \sum_i \; C_i(\mu) \; \langle (\pi\,\pi) \,|\, Q_i(\mu) \,|\, K \rangle_{I=2}$$

Ω_{IB} = 0.25± 0.08 (Munich from Buras & Gerard)
0.25± 0.15 (Rome Group)   0.16± 0.03 (Ecker et al.)
0.10± 0.20 Gardner & Valencia, Maltman & Wolf, Cirigliano & al.

$$A^{I=0,2}_i(\mu) = \langle (\pi\,\pi)_{I=0,2} | Q_i(\mu) | K \rangle$$
$$= Z_{ik}(\mu\, a) \langle (\pi\,\pi)_{I=0,2} | Q_k(a) | K \rangle$$

Where $Q_i(a)$ is the bare lattice operator
And a the lattice spacing.

The effective Hamiltonian can then be read as:

$$\langle F | H^{\Delta S=1} | I \rangle = G_F/\sqrt{2} V_{ud} V_{us}^* \Sigma_i C_i(1/a) \langle F | Q_i(a) | I \rangle$$

In practice the renormalization scale (or 1/a) are the scales which separate short and long distance dynamics

GENERAL FRAMEWORK

$$\langle H^{\Delta S=1} \rangle = G_F/\sqrt{2}\, V_{ud}\, V_{us}^{*} \ldots \Sigma_i\, C_i\, (\mathbf{a})\, \langle Q_i(\mathbf{a}) \rangle$$

$M_W = 100$ GeV

perturbative regime

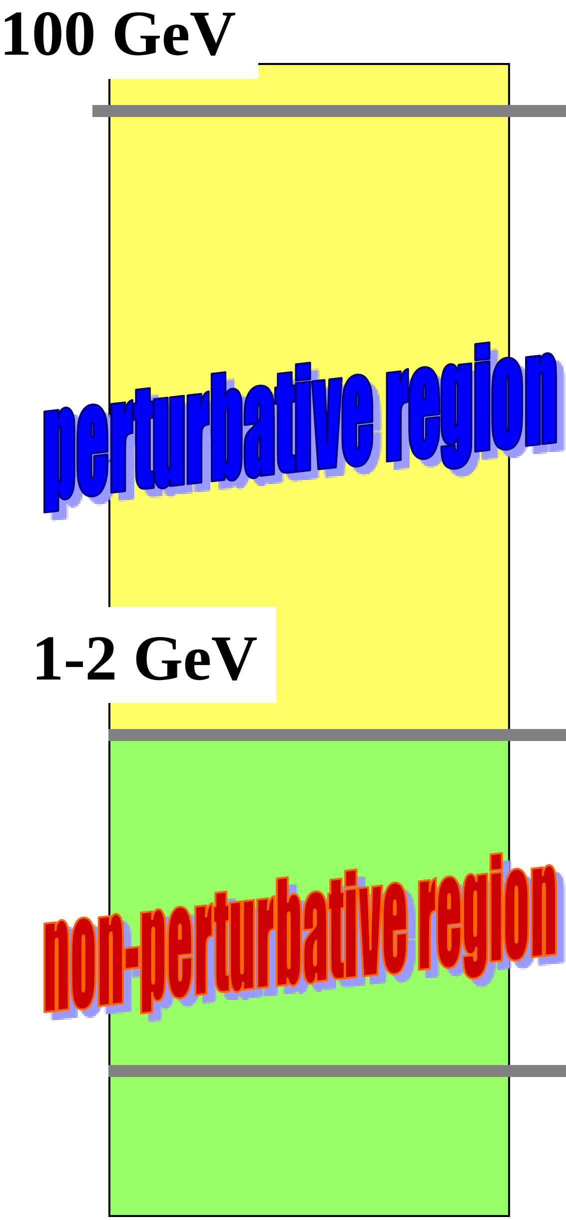
Effective Theory - quark & gluons

$a^{-1} = 2\text{-}5$ GeV

Hadronic non-perturbative region

Chiral regime

$\Lambda_{QCD}, M_K = \mathbf{0.2\text{-}0.5\ GeV}$



Large mass scale: heavy degrees of freedom (m_t , $M_{W,}$ M_s) are removed and their effect included in the Wilson coefficients

renormalizazion scale μ (inverse lattice spacing 1/a); this is the scale where the quark theory is matched to the effective hadronic theory

Scale of the low energy process $\Lambda \ll M_W$

THE SCALE PROBLEM: Effective theories prefer low scales, Perturbation Theory prefers large scales

if the scale μ is too low
problems from higher dimensional operators

(Cirigliano, Donoghue, Golowich)

- it is illusory to think that the problem is solved by using dimensional regularization

on the lattice this problem is called

DISCRETIZATION ERRORS

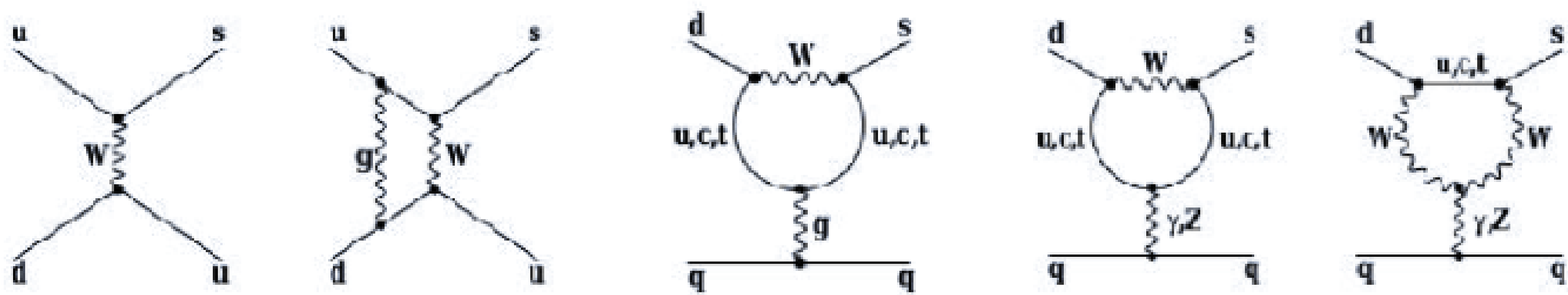
(reduced by using improved actions and/or scales $\mu > 2\text{-}4$ GeV

Weak Hamiltonian for $K \to \pi\pi$

Weak Hamiltonian is given by local four-quark operator *Courtesy by Xu Feng*

$$\mathcal{H}^{\Delta S=1} = \frac{G_F}{\sqrt{2}} V_{ud} V_{us}^* \left\{ \sum_{i=1}^{10} [z_i(\mu) + \tau y_i(\mu)]\, Q_i \right\}, \quad \tau = -\frac{V_{td} V_{ts}^*}{V_{ud} V_{us}^*}$$

- $\tau = -\frac{V_{td} V_{ts}^*}{V_{ud} V_{us}^*} = 1.543 + 0.635i$
- $z_i(\mu)$ and $y_i(\mu)$ are perturbative Wilson coefficients
- Q_i are local four-quark operator



Current-current operator Q_1, Q_2 dominate $\mathrm{Re}[A_0]$, $\mathrm{Re}[A_2]$

QCD penguin $Q_3 - Q_6$ Q_6 dominate $\mathrm{Im}[A_0]$

Electro-weak penguin $Q_7 - Q_{10}$ Q_7, Q_8 dominate $\mathrm{Im}[A_2]$

New local four-fermion operators are generated

$Q_1 = (\bar{s}_L{}^A \gamma_\mu u_L{}^B)(\bar{u}_L{}^B \gamma_\mu d_L{}^A)$ Current-Current

$Q_2 = (\bar{s}_L{}^A \gamma_\mu u_L{}^A)(\bar{u}_L{}^B \gamma_\mu d_L{}^B)$

$Q_{3,5} = (\bar{s}_R{}^A \gamma_\mu d_L{}^A)\sum_q (\bar{q}_{L,R}{}^B \gamma_\mu q_{L,R}{}^B)$ Gluon

$Q_{4,6} = (\bar{s}_R{}^A \gamma_\mu d_L{}^B)\sum_q (\bar{q}_{L,R}{}^B \gamma_\mu q_{L,R}{}^A)$ Penguins

$Q_{7,9} = 3/2(\bar{s}_R{}^A \gamma_\mu d_L{}^A)\sum_q e_q (\bar{q}_{R,L}{}^B \gamma_\mu q_{R,L}{}^B)$ Electroweak

$Q_{8,10} = 3/2(s_R{}^A \gamma_\mu d_L{}^B)\sum_q e_q (q_{R,L}{}^B \gamma_\mu q_{R,L}{}^A)$ Penguins

+ Chromomagnetic and electromagnetic operators

$$\mathcal{A}(K \to \pi\pi) = \sum_i C_W^i(\mu) \langle \pi\pi | O_i(\mu) | K \rangle$$

$\not{C}\not{P}$ Violation in the Neutral Kaon System

Expanding in several "small" quantities

$$\eta^{00} = \frac{\langle \pi^0\pi^0 | \mathcal{H}_W | K_L \rangle}{\langle \pi^0\pi^0 | \mathcal{H}_W | K_S \rangle} \sim \varepsilon - 2\,\varepsilon'$$

$$\eta^{+-} = \frac{\langle \pi^+\pi^- | \mathcal{H}_W | K_L \rangle}{\langle \pi^+\pi^- | \mathcal{H}_W | K_S \rangle} \sim \varepsilon + \varepsilon'$$

Conventionally:

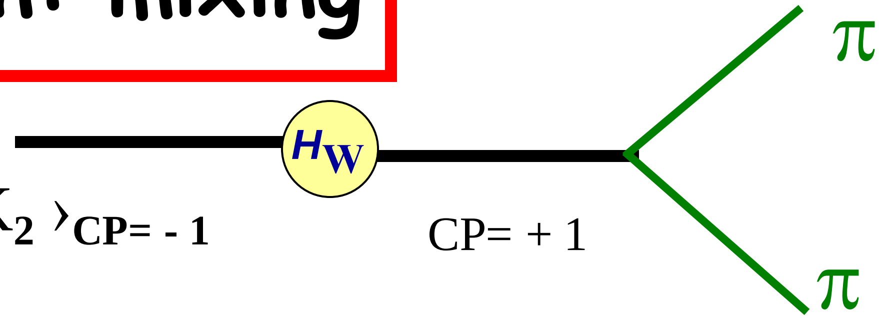
$$| K_S \rangle = | K_1 \rangle_{CP=+1} + \varepsilon \, | K_2 \rangle_{CP=-1}$$

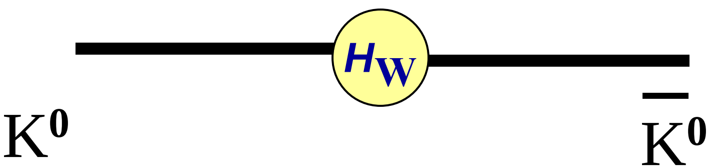
$$| K_L \rangle = | K_2 \rangle_{CP=-1} + \varepsilon \, | K_1 \rangle_{CP=+1}$$

Indirect CP violation: mixing

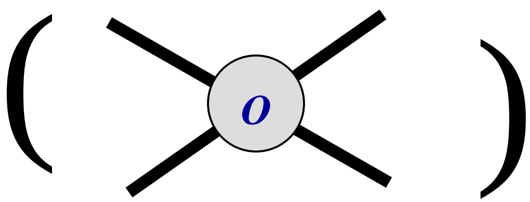
ε_K

$|K_L\rangle = |K_2\rangle_{CP=-1}$

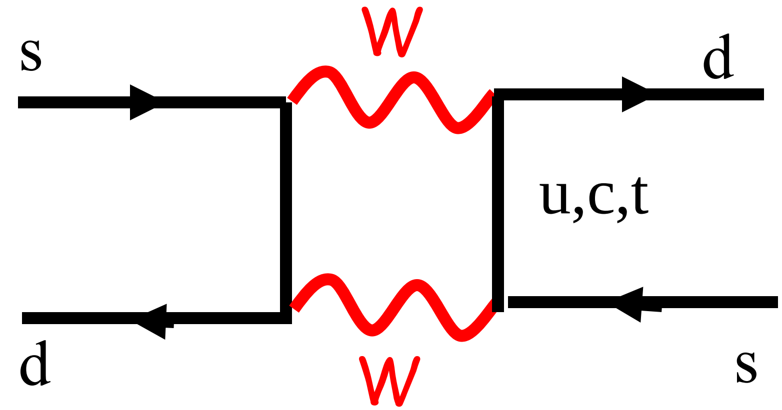




ΔS=2



Complex ΔS=2 effective coupling



Box diagrams:
They are also responsible for B^0 - $\bar{B}^0$ mixing

$\Delta m_{d,s}$

$$|\varepsilon_K| \sim C_\varepsilon \, A^2 \lambda^6 \sigma \sin\delta \{F(x_c, x_t) + F(x_t)[A^2 \lambda^4 (1 - \sigma\cos\delta)] - F(x_c)\} B_K$$

Inami-Lin Functions + QCD Corrections (NLO)

$$\eta = \sigma \sin\delta \quad \rho = \sigma \cos\delta$$

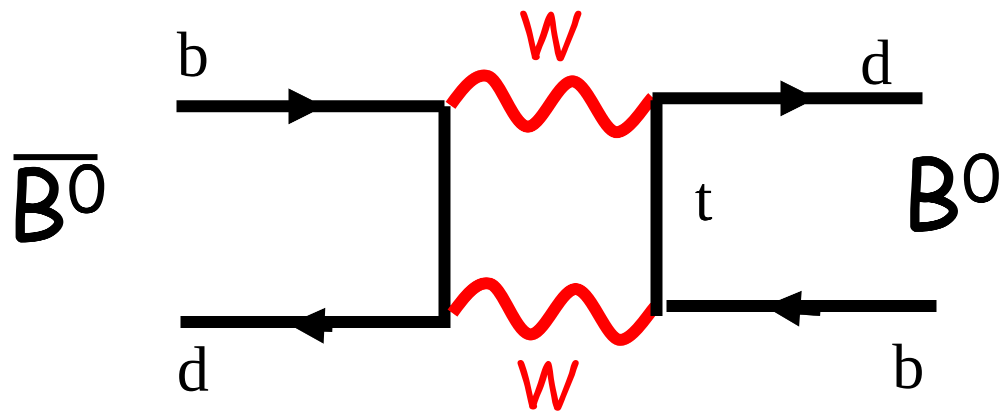
$$C_\varepsilon = \frac{G_F^2 M_W^2 M_K f_K^2}{6\sqrt{2}\,\pi^2 \Delta M_K}$$

$$\langle \overline{K}^0 | (\bar{s}\gamma_\mu (1 - \gamma_5) d)^2 | K^0 \rangle = 8/3 \, f_K^2 M_K^2 B_K$$

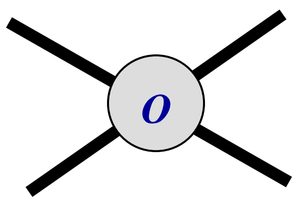
B^0 - B^0 mixing

ΔB=2 Transitions

$$H_{eff}^{\Delta B=2} = \begin{pmatrix} H_{11} & H_{12} \\ H_{21} & H_{22} \end{pmatrix}$$



$$H_{eff}^{\Delta B=2} =$$



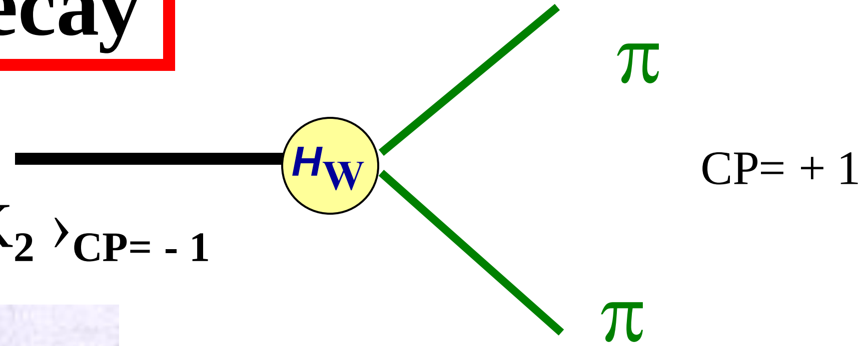
$$\propto \quad (\overline{\mathbf{d}}\,\gamma_\mu\,(\mathbf{1} - \gamma_5)\,\mathbf{b})^2$$

Hadronic matrix element

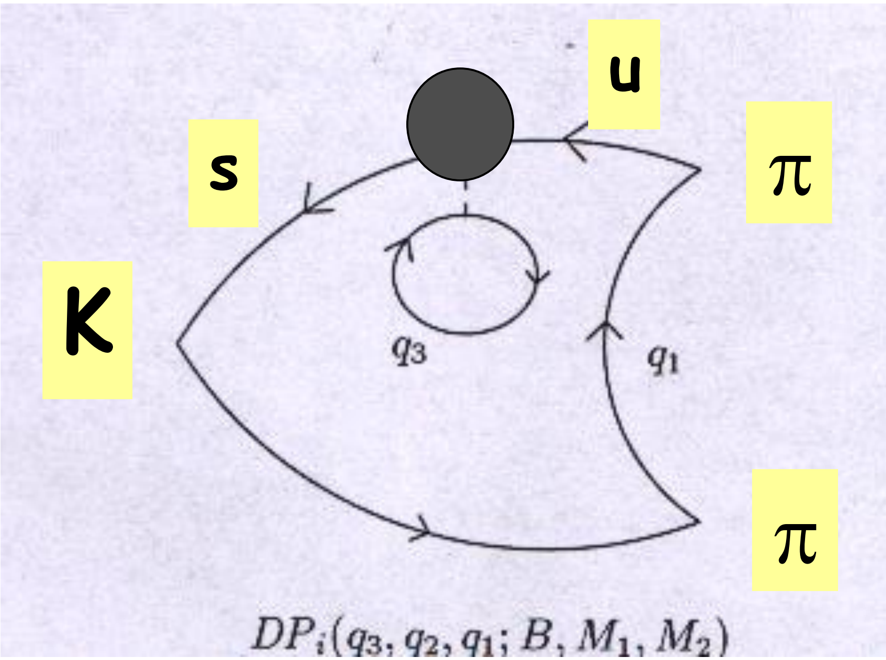
CKM

$$\Delta m_{d,s} = \frac{G^2_F\ M^2_W}{16\ \pi^2}\ A^2\ \lambda^6\ F_{tt}\left(\frac{m^2_t}{M^2_W}\right) \langle\ O\ \rangle$$

Direct CP violation: decay



$| K_L \rangle = | K_2 \rangle_{CP= - 1}$



Complex ΔS=1 effective coupling

$$L^{\not{CP}} = L^{\Delta F=0} + L^{\Delta F=1} + L^{\Delta F=2}$$

ΔF=0 $d_e < 1.5\ 10^{-27}$ e cm $d_N < 6.3\ 10^{-26}$ e cm

ΔF=1 $\varepsilon' / \varepsilon$
+ B decays (see later)

ΔF=2 ε and $B \rightarrow J/\psi\, K_s$

The extraordinary progress of the experimental measurements requires accurate theoretical predictions

Precision flavor physics requires the control of hadronic effects for which lattice QCD simulations are essential

$$Q^{EXP} = V_{CKM} \langle F | \hat{O} | I \rangle$$

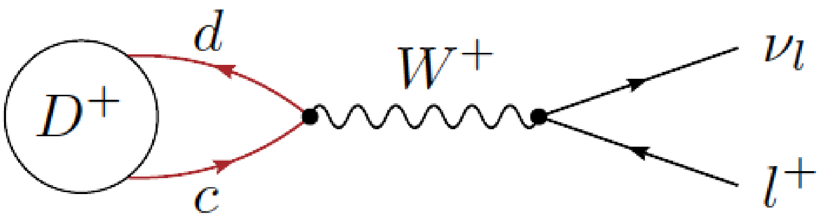
SM

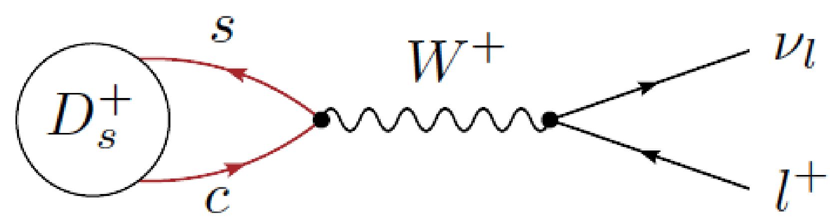
$$Q^{EXP} = \sum_i C^i_{SM}(M_W, m_t, \alpha_s) \langle F | \hat{O}_i | I \rangle + \sum_{i'} C^{i'}_{Beyond}(\tilde{m}_\beta, \alpha_s) \langle F | \hat{O}_{i'} | I \rangle$$

BSM

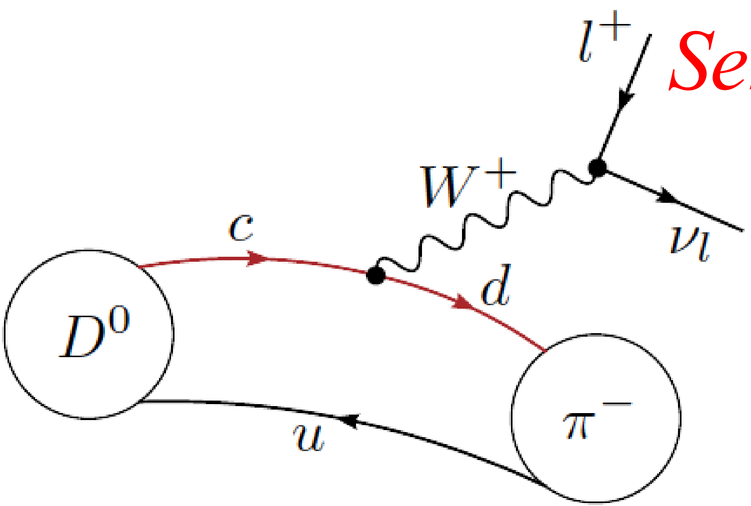
What can be computed and what cannot be computed

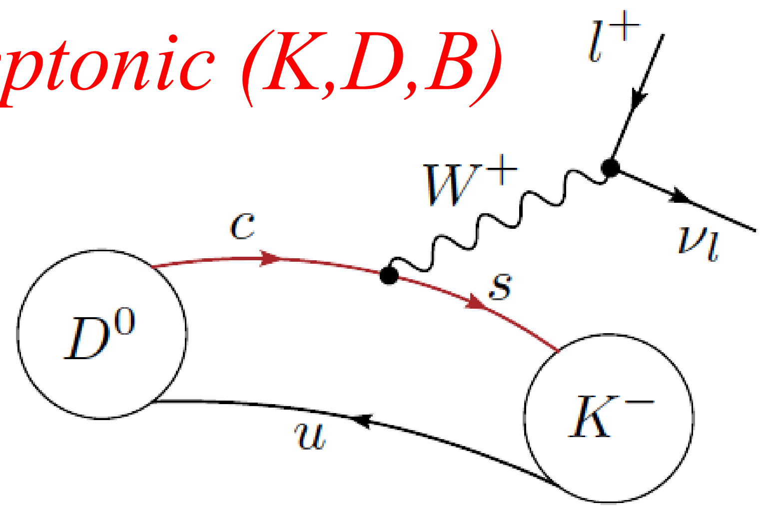
Leptonic (π,K,D,B)





Semileptonic (K,D,B)

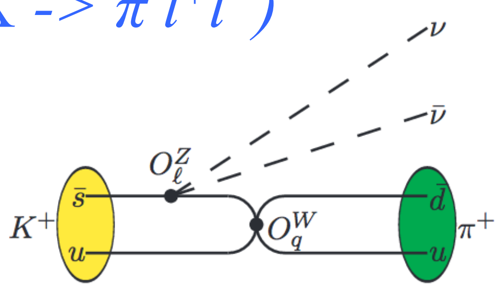




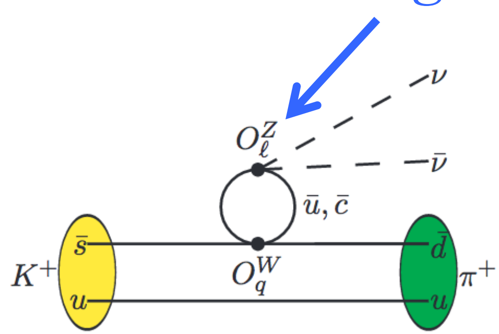
(some) Radiative and Rare

(also $K \to \pi\, l^+ l^-$)

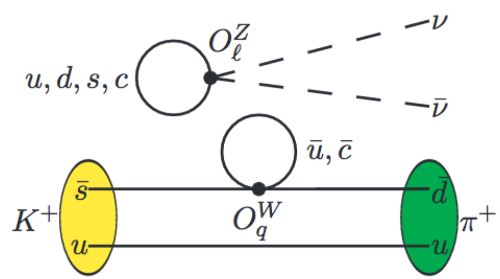
long distance effects



connected diag.



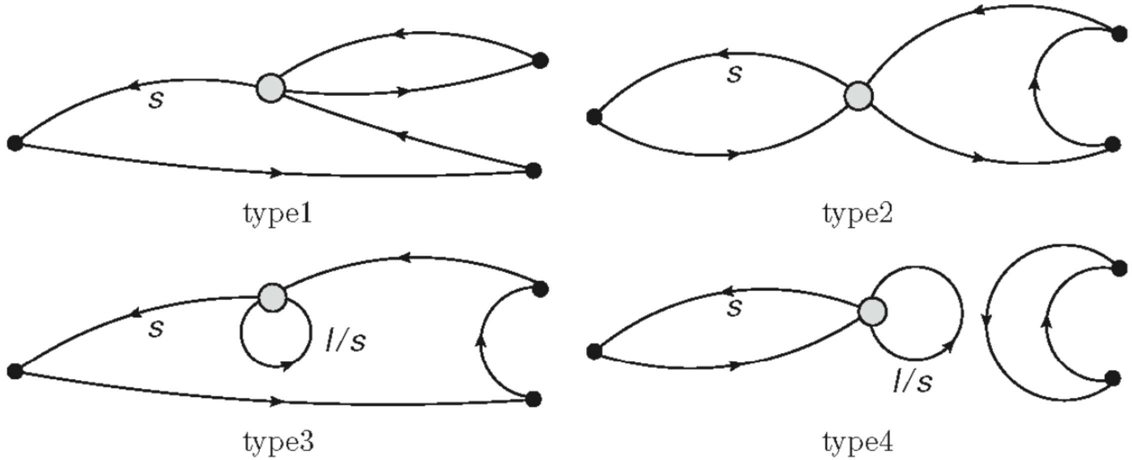
self-loop diag.



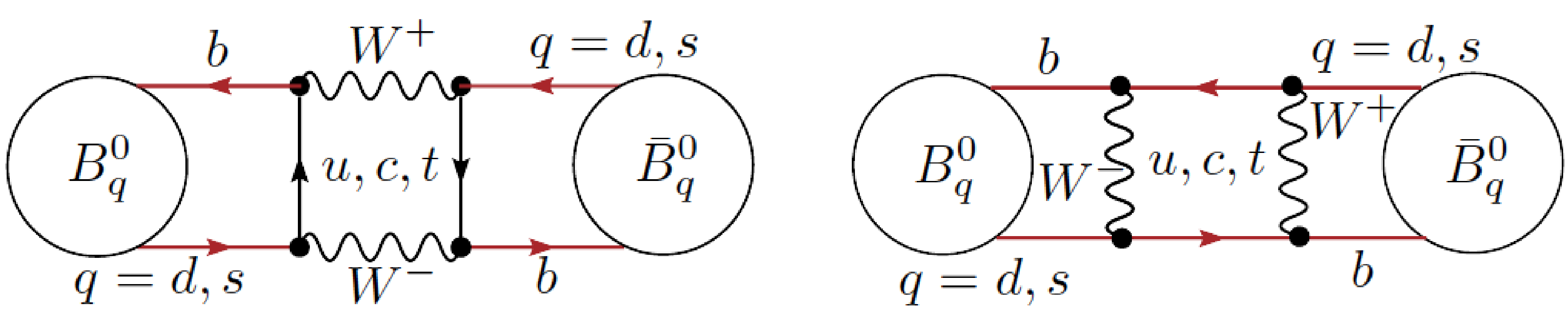
disconnected diag.

Non-leptonic *B -> ππ,Kπ, etc. No !*

but only below the inelastic threshold (may be also 3 body decays)



Neutral meson mixing (local)

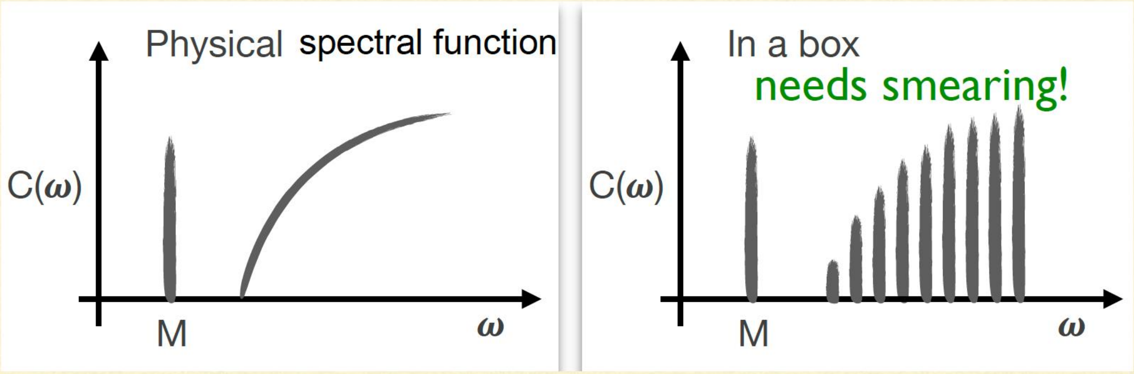


+ some long distance contributions to K and D neutral meson mixing + short distance contributions to B-> $K^{(*)}$ l^+l^-

INCLUSIVE DECAYS ON THE LATTICE

Inclusive processes impractical to treat directly on the lattice. Vacuum current correlators computed in euclidean space-time are related to $e^+e^- \to$ hadrons or τ decay via analyticity. In our case the correlators have to be computed in the B meson, but analytic continuation more complicated: two cuts, decay occurs only on a portion of the physical cut.

While the lattice calculation of the spectral density of hadronic correlators is an ***ill-posed problem***, the spectral density is accessible after smearing
Hansen, Meyer, Robaina, Hansen, Lupo, Tantalo, Bailas, Hashimoto, Ishikawa

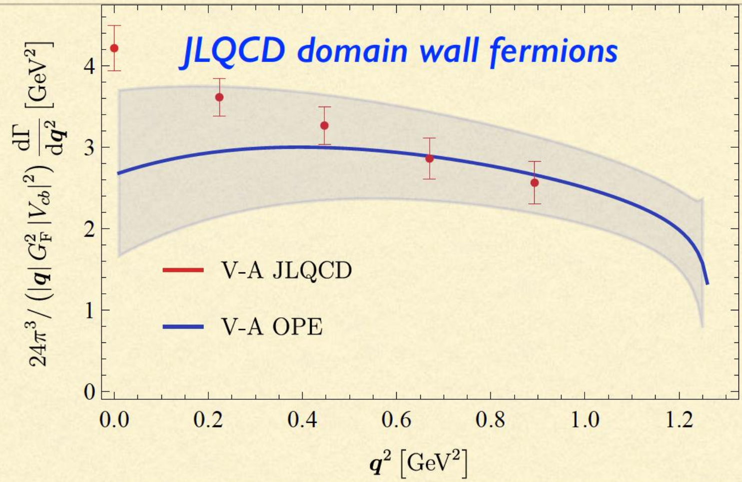


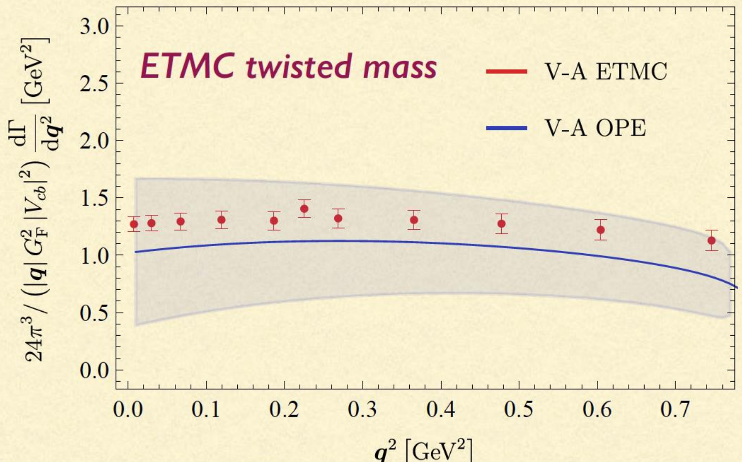
W. Jay @Snowmass workshop

courtesy of P. Gambino

PG, Hashimoto, Maechler, Panero, Sanfilippo, Simula, Smecca, Tantalo, 2203.11762

LATTICE VS OPE





m_b^{kin} (JLQCD)	2.70 ± 0.04
$\overline{m}_c$(2 GeV) (JLQCD)	1.10 ± 0.02
m_b^{kin} (ETMC)	2.39 ± 0.08
$\overline{m}_c$(2 GeV) (ETMC)	1.19 ± 0.04
μ_π^2	0.57 ± 0.15
ρ_D^3	0.22 ± 0.06
$\mu_G^2(m_b)$	0.37 ± 0.10
ρ_{LS}^3	-0.13 ± 0.10
$\alpha_s^{(4)}$(2 GeV)	0.301 ± 0.006

OPE inputs from fits to exp data (physical m_b), HQE of meson masses on lattice

1704.06105, J.Phys.Conf.Ser. 1137 (2019) 1, 012005

We include $O(1/m_b^3)$ and $O(\alpha_s)$ terms

Hard scale $\sqrt{m_c^2 + \mathbf{q}^2} \sim 1-1.5\,\mathrm{GeV}$

We do not expect OPE to work at high $|\mathbf{q}|$

Twisted boundary conditions allow for any value of $\vec{q}^{\,2}$

Smaller statistical uncertainties

courtesy of P. Gambino

Evaluating hadronic amplitudes on the lattice through the spectral representation

Giuseppe Gagliardi, INFN Sezione di Roma Tre

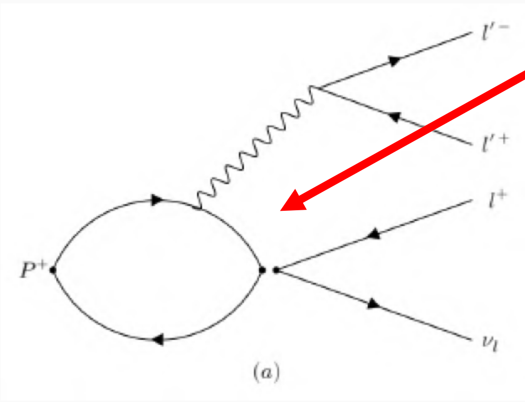
R. Frezzotti, V. Lubicz, ~~G. Martinelli~~, F. Mazzetti, C.T. Sachrajda,
F. Sanfilippo, S. Simula, N. Tantalo

ETMC meeting, 8-10 February 2023, Bern.

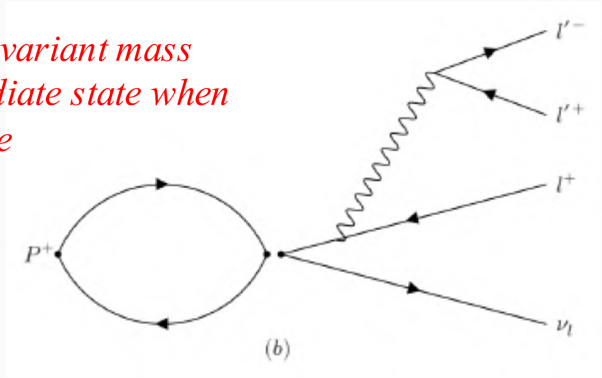
Radiative decays

$D_s^{\pm} \to l'^{+} l'^{-} l^{\pm} \nu_l$ decays

The $P^+ \equiv \bar{D}\gamma^5 U \to l'^+ l'^- l^+ \nu_l$ decays



Large invariant mass intermediate state when q^2 large



- Diagram (b) is perturbative, only QCD input is decay constant f_P.
- Diagram (a) is non-perturbative. Virtual photon γ^* emitted from either a $U-$type or a $D-$type quark line. For $P^+ = D_s^+$: $U = c$, $D = s$.

Non-perturbative QCD contribution encoded in the hadronic tensor

$$H_W^{\mu\nu}(k, \boldsymbol{p}) = \int d^4x\, e^{ik\cdot x} \langle 0 | T[J_{\rm em}^{\mu}(x) J_W^{\nu}(0)] | P(\boldsymbol{p}) \rangle, \quad W = V, A$$

- $k = (E_\gamma, \boldsymbol{k})$ is photon 4-momentum, $\boldsymbol{p}$ is P-meson 3-momentum.
- We neglect $\mathrm{SU}(3)$-vanishing quark-line disconnected diagrams.

sin 2β is measured directly from $B \rightarrow J/\psi K_s$ decays at Babar & Belle

$$A_{J/\psi K_s} = \frac{\Gamma(B_d^0 \rightarrow J/\psi K_s, t) - \bar{\Gamma}(B_d^0 \rightarrow J/\psi K_s, t)}{\Gamma(B_d^0 \rightarrow J/\psi K_s, t) + \bar{\Gamma}(B_d^0 \rightarrow J/\psi K_s, t)}$$

$$A_{J/\psi K_s} = \sin 2\beta \;\; \sin(\Delta m_d t)$$

DIFFERENT LEVELS OF THEORETICAL UNCERTAINTIES (STRONG INTERACTIONS)

1) First class quantities, with reduced or negligible uncertainties

$A_{CP}(B \to J/\psi K_s)$ γ *from* $B \to DK$

$K^0 \to \pi^0 \nu \bar{\nu}$

2) Second class quantities, with theoretical errors of O(10%) or less that can be reliably estimated

ε_K $\Delta M_{d,s}$

$\Gamma(B \to c, u)$, $K^+ \to \pi^+ \nu\bar{\nu}$

3) Third class quantities, for which theoretical predictions are model dependent (BBNS, charming, etc.)

In case of discrepacies we cannot tell whether is *new physics or we must blame the model*

$B \to K\pi$ $B \to \pi^0 \pi^0$

$B \to \phi K_s$

Flavour Physics

__1963:__ Cabibbo Angle
__1964:__ CP violation in K decays *
__1970__ GIM Mechanism
__1973:__ CP Violation needs at least three quark families (CKM) *
__1975:__ discovery of the tau lepton – 3rd lepton family *
__1977:__ discovery of the b quark - 3rd quark family *
__2003/4:__ CP violation in B meson decays

** Nobel Prize*

Discoveries from Flavor Physics

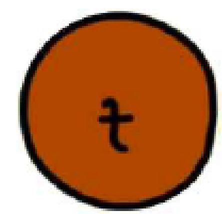
CP Violation !!

- the tiny branching ratio of the decay $K_L \to \mu^+\mu^-$ led to the prediction of the charm quark to suppress FCNCs
  (Glashow, Iliopoulos, Maiani 1970)

- the measurement of the frequency of kaon anti-kaon oscillations allowed a successful prediction of the charm quark mass
  (Gaillard, Lee 1974)

(direct discovery of the charm quark in 1974 at SLAC and BNL)

- the observation of CP violation in kaon anti-kaon oscillations led to the prediction of the 3rd generation of quarks
  (Kobayashi, Maskawa 1973)



- the measurement of the frequency of B - $\bar{B}$ oscillations allowed to predict the large top quark mass
  (various authors in the late 80's)

(direct discovery of the bottom quark in 1977 at Fermilab)
(direct discovery of the top quark in 1995 at Fermilab)

the Standard Model and beyond

Vacuum Energy

Hierarchy

Vacuum Stability

$$\mathcal{L} = \Lambda^4 + \Lambda^2 H^2 + \lambda H^4 +$$

$$(D_\mu H)^2 + \bar{\psi} \not{D} \psi + F_{\mu\nu}^2 + F_{\mu\nu} \tilde{F}_{\mu\nu}$$

Strong ~~CP~~

$$Y H \bar{\psi} \psi + \frac{1}{\Lambda} (\bar{L} H)^2 + \frac{1}{\Lambda^2} \sum_i C_i O_i + \ldots$$

Flavor puzzle

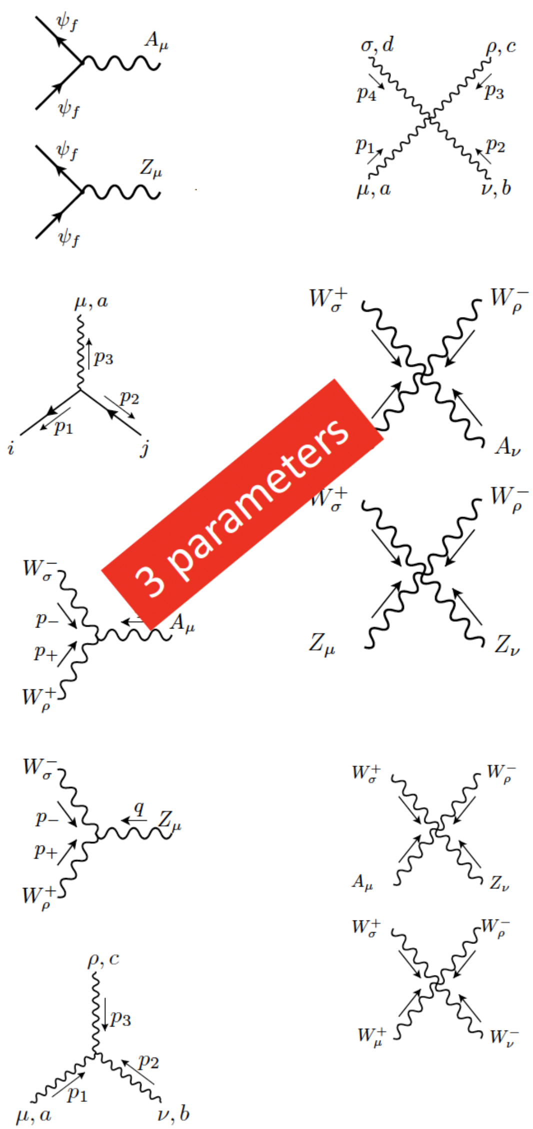
Neutrino Masses

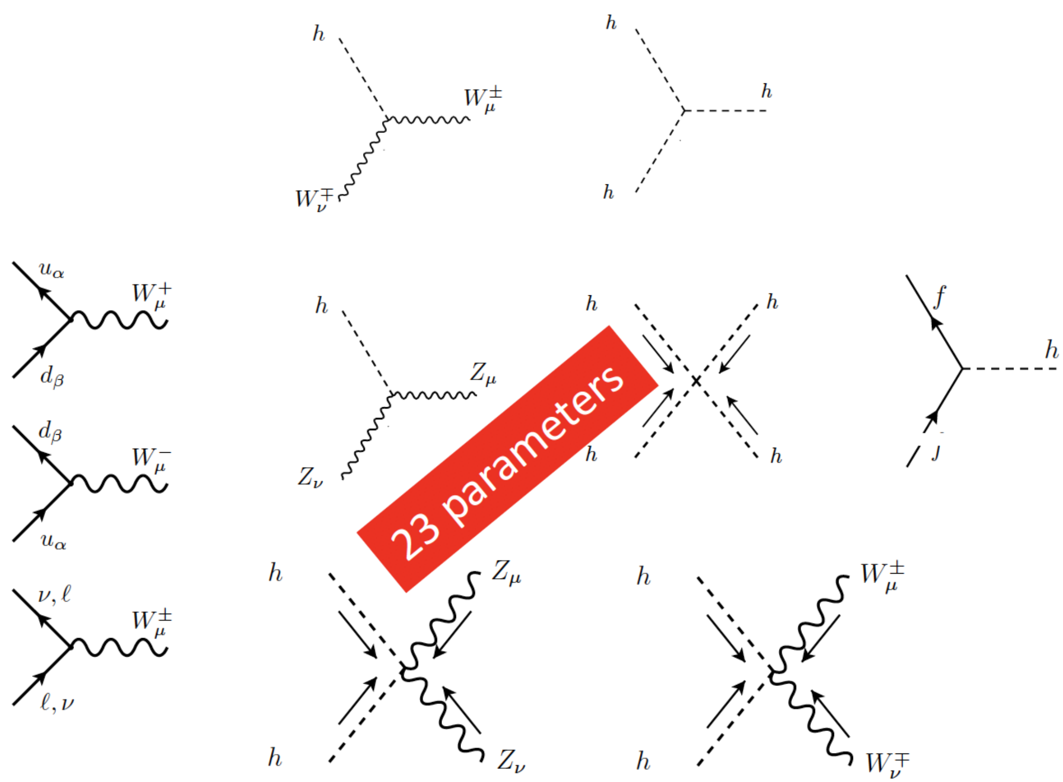
New Physics
Possible breaking of accidental symmetries

The Standard Model

$SU(3) \times SU(2) \times U(1)_Y$

Higgs + gauge principle





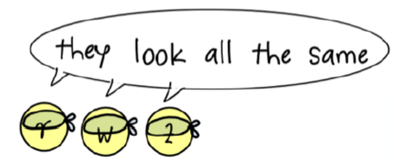
from elegance to caos !!

If we are looking for the suspect that could be hiding some secret obviously the higgs is the one!

Hints of NP structure: Flavor symmetries of the SM

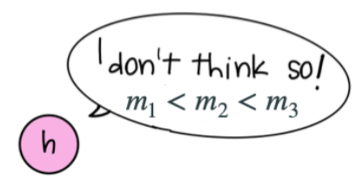
- Standard Model (SM) gauge sector is *flavor blind!*

$$\mathscr{G}_F(\text{SM}) = U(3)^5 \equiv U(3)_q \times U(3)_u \times U(3)_d \times U(3)_\ell \times U(3)_e$$



Turn on Yukawas

$$Y_{ij} \bar{\Psi}^i_L H \Psi^j_R$$



$$\mathscr{G}_F(\text{SM}) = U(1)_B \times U(1)_L$$

courtesy of B.A. Stefanek

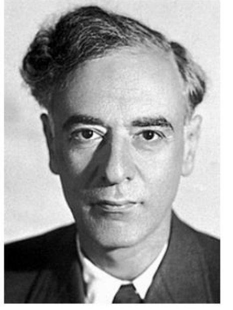
The Weirdness of the Standard Model

- Three families

"who ordered that ?" I. Rabi

- Fundamental breaking of Parity

"space cannot be asymmetric!" L. Landau



$3m_\nu + (2+1)\delta_{PMNS} + 3\theta_{PMNS} = 9$

- Predictivity: 3 gauge couplings+ 16 higgs couplings (+ 7 higgs-neutrino) !

  + the coupling θ of strong CP violation

"has too many arbitrary features for [its] predictions to be taken very seriously" S. Weinberg '67

$$3g_i + (\lambda, M_H) + 6m_q + 3m_\ell + \delta + 3\theta_{CKM} + \theta_{QCD} = 19$$

electromagnetic **neutral currents** **charged currents**

$$\mathcal{L}_{int} = -eA^{\mu} J_{\mu}^{em} - \frac{g_W}{2\cos\theta_W} Z^{\mu} J_{\mu}^{Z} - \frac{g_W}{2\sqrt{2}} [W^{\mu} (J^{W})_{\mu}^{\dagger} + h.c.]$$

$$J_{\mu}^{Z} = 2J_{\mu}^{3} - 2\sin^2\theta_W J_{\mu}^{em}$$

$$W_{\mu}(J_W^{\dagger})^{\mu} + h.c. = W_{\mu}^{-}\,\bar{u}\gamma^{\mu}(1-\gamma_5)d + W_{\mu}^{+}\,\bar{d}\gamma^{\mu}(1-\gamma_5)u$$

for Hemiticity $g_1=g_2$

$$\mathcal{L} = g_1\, W_\mu^-\, \bar{u}\gamma^\mu(1-\gamma_5)d + g_2\, W_\mu^+\, \bar{d}\gamma^\mu(1-\gamma_5)u \qquad g_1 \neq g_2$$

$$[W_\mu^-\, \bar{u}\gamma^\mu(1-\gamma_5)d]^\dagger = W_\mu^+\, d^\dagger\, [(\gamma^\mu)^\dagger(\gamma^0)^\dagger - (\gamma_5)^\dagger(\gamma^\mu)^\dagger(\gamma^0)^\dagger]u = W_\mu^+\, d^\dagger\, [\gamma^0(\gamma^0(\gamma^\mu)^\dagger\gamma^0) - \gamma_5\gamma^0(\gamma^0(\gamma^\mu)^\dagger\gamma^0)]u$$

$$(\gamma^0(\gamma^\mu)^\dagger\gamma^0) = \gamma^\mu \qquad = W_\mu^+\, \bar{d}[\gamma^\mu) - \gamma^\mu\gamma_5]u$$

$$\mathcal{L} = g_W\left[W_\mu^-\, \bar{u}\gamma^\mu(1-\gamma_5)d + W_\mu^+\, \bar{d}\gamma^\mu(1-\gamma_5)u\right]$$

hermiticity

$$P^\dagger \mathcal{L} P = g_W\left[W_\mu^-\, \bar{u}\gamma^\mu(1+\gamma_5)d + W_\mu^+\, \bar{d}\gamma^\mu(1+\gamma_5)u\right]$$

parity violation

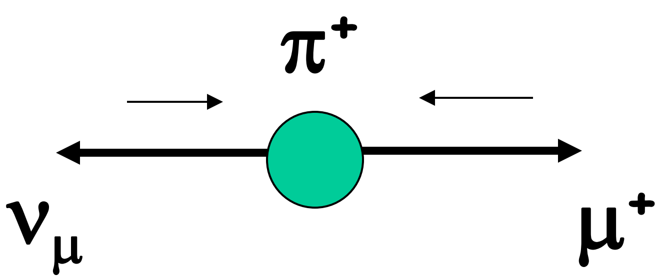
$$C^\dagger \mathcal{L} C = g_W\left[W_\mu^+\, \bar{d}\gamma^\mu(1+\gamma_5)u + W_\mu^-\, \bar{u}\gamma^\mu(1+\gamma_5)d\right]$$

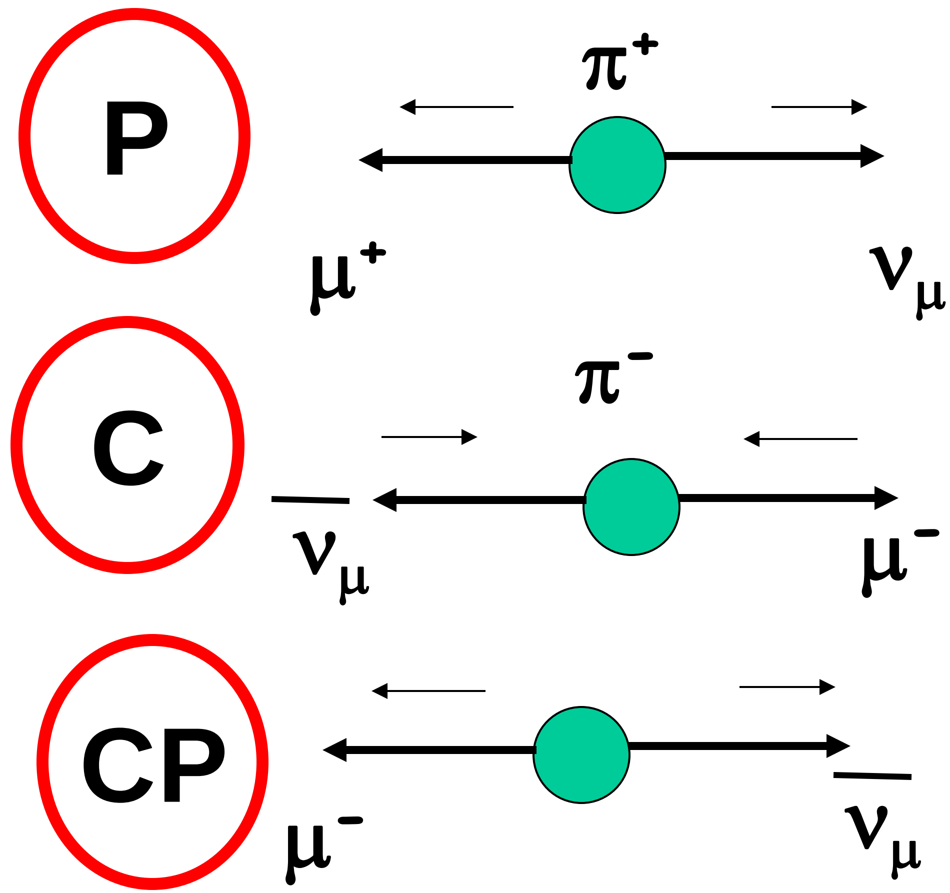
charge conj. violation

$$\boxed{(CP)^\dagger \mathcal{L} (CP) = \mathcal{L}}$$

CP conserved

Relativistic Quantum Mechanics → Antimatter CPT Theorem





CP Violation was discovered about 37 years ago in $K^0 - \bar{K}^0$ mixing (weak interactions)

In the Standard Model the quark mass matrix, from which the CKM Matrix and $\not{CP}$ originate, is determined by the Yukawa Lagrangian which couples fermions and Higgs

$$\mathcal{L}^{quarks} = \mathcal{L}^{kinetic} + \mathcal{L}^{weak\ int} + \mathcal{L}^{yukawa}$$

CP invariant

$\not{CP}$ and symmetry breaking are closely related !

In 1967 Andrei Sakharov pointed out that, for the universe to evolve from the initial matter-antimatter fireball to the present matter antimatter asymmetric state, 4 conditions must be fulfilled:

1) Baryon number violation $\Delta B \neq 0$ (GUT ??)

$$e^+ + \bar{d} \rightarrow X \rightarrow u + u \qquad (\Delta(B-L) = 0)$$

Lepton number violation is possible but not necessary and could be zero

2) Charge symmetry violation $\not{C}$

$$\Gamma(e^+ + \bar{d} \rightarrow X \rightarrow u + u) \neq \Gamma(e^- + d \rightarrow X \rightarrow \bar{u} + \bar{u})$$

3) $\not{CP}$ violation: the number of left handed up quarks produced by X must be different from the number of right handed up antiquarks

4) The universe was not in equilibrium when this happened, otherwise if

$\Gamma(e^+ + d \to u + u) > \Gamma(e^- + d \to \bar{u} + \bar{u})$ then

$\Gamma(u + u \to e^+ + d) > \Gamma(\bar{u} + \bar{u} \to e^- + d)$

$$\langle B \rangle = Tr[e^{-\beta H} B] = Tr[(CPT)(CPT)^{-1} e^{-\beta H} B]$$

$$= Tr[e^{-\beta H} (CPT)^{-1} B (CPT)] = -\langle B \rangle$$

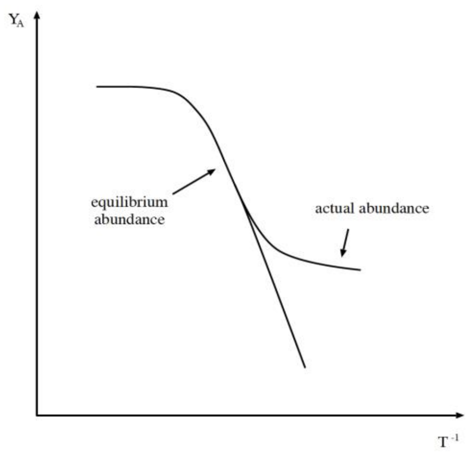


Figure 2: The behaviour of $Y_A \equiv n_A/s$ as a function of decreasing temperature for a massive, (non)relativistic particle species A falling out of thermal equilibrium.

$$\langle B(t) \rangle = \langle B(0) \rangle$$

Neutrino Decoupling nuCB

Photon Decoupling CMB

Nucleosynthesis light elements

Wimp (?) BAU

$$n^0 \longrightarrow p^+ + e^- + \bar{\nu}_e \qquad p^+ + n^0 \longrightarrow {}^2_1D + \gamma$$

$${}^2_1D + p^+ \longrightarrow {}^3_2He + \gamma \qquad {}^2_1D + {}^2_1D \longrightarrow {}^3_2He + n^0$$

$${}^2_1D + {}^2_1D \longrightarrow {}^3_1T + p^+ \qquad {}^3_1T + {}^2_1D \longrightarrow {}^4_2He + n^0$$

$${}^3_2He + {}^4_2He \longrightarrow {}^7_3Li + \gamma \qquad {}^3_2He + n^0 \longrightarrow {}^3_1T + p^+$$

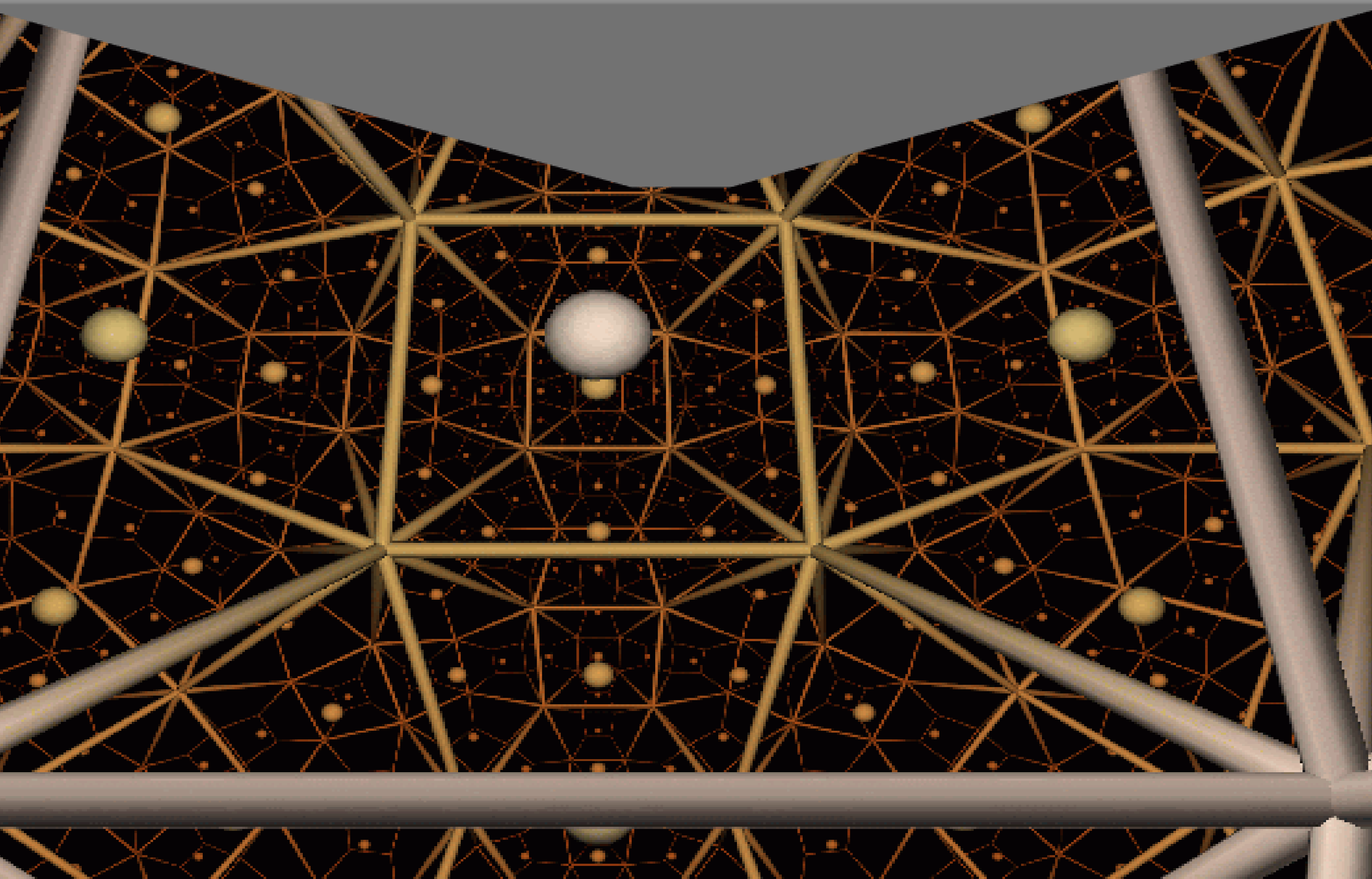
$${}^3_2He + {}^2_1D \longrightarrow {}^4_2He + p^+ \qquad {}^3_2He + {}^4_2He \longrightarrow {}^7_4Be + \gamma$$

$${}^7_3Li + p^+ \longrightarrow {}^4_2He + {}^4_2He \qquad {}^7_4Be + n^0 \longrightarrow {}^7_3Li + p^+$$

Primordial Nucleosynthesis

Big Bang Nucleosynthesis (BBN) began approximately 1 second into the Big Bang and lasts for about 3 minutes. BBN is the only window into the conditions of the early universe before the CMB. During this epoch, the temperature is optimal for the formation of light nuclei, resulting in the synthesis of D,^{3}He,^{4}He, and ^{7}Li. The story begins with all of the energy in the universe condensed into a single point. Suddenly, it exploded outwards and the universe was born. As it expanded, it cooled and began to form matter.

The amount of $\not{CP}$, discovered in 1964 in mixing (see below) is however too small to explain the scarcity of antimatter in the universe.



Baryon Number Violation & CP Violation in the Standard Model

$$\partial_\mu J^\mu_B = \frac{\partial \rho_B}{\partial t} - \vec{\nabla} \cdot \vec{J}_B = \frac{N_f}{32\pi^2}\left(g_W^2 W^a_{\mu\nu} \tilde{W}_a^{\mu\nu} - g_Y^2 B_{\mu\nu} \tilde{B}^{\mu\nu}\right)$$

classically

$$\frac{dQ_B}{dt} = 0 \qquad Q_B = \int d^3x\, \rho_B(\vec{x}, y)$$

quantum & non perturbative effects

$$\Gamma \sim e^{-16\pi^2/g_W^2} \sim e^{-165}$$

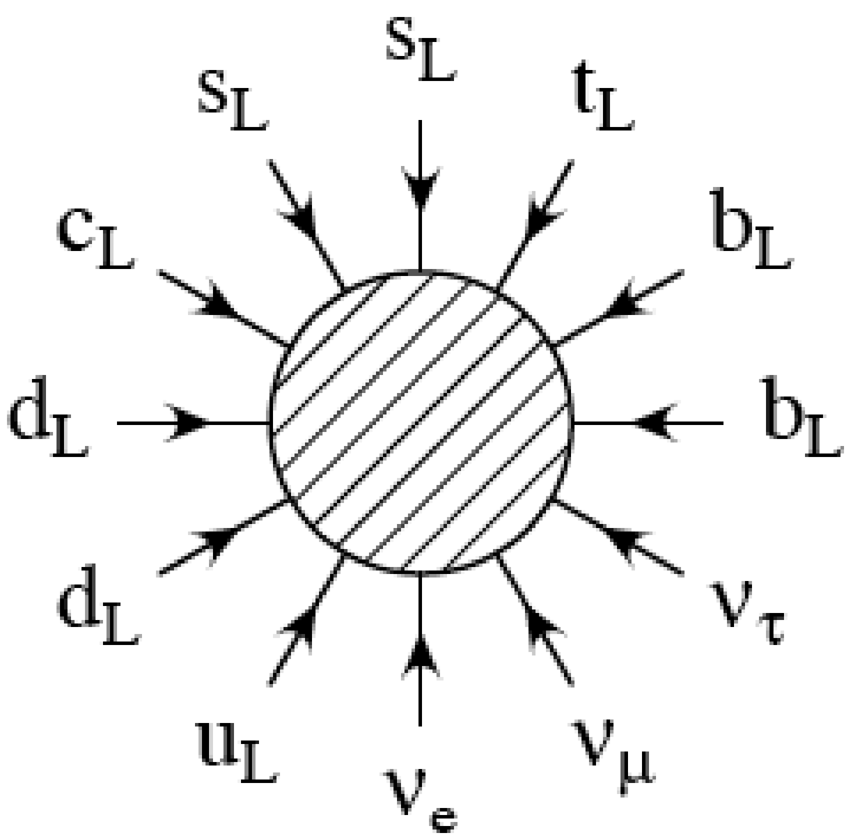
T=0

T >> E_{EW}

$$\frac{\Gamma_{B+L}}{V} \sim \alpha_{em}^5 \log \alpha_{em}^{-1}\, T^4$$

$$T \gg T_{ew} \sim 250\, GeV \qquad \alpha_{em} \sim \frac{1}{137, \ldots.}$$

$$u + d \to \bar{d} + 2\bar{s} + \bar{c} + 2\bar{b} + \bar{t} + \bar{\nu}_e + \bar{\nu}_\mu + \bar{\nu}_\tau \,. \tag{49}$$



Weak Couplings, Quark Masses and CP Violation in the Standard Model

Some General Consideration:

Discrete Symmetries C P T:

1) Charge Conjugation
2) Parity
3) Time reversal

All violated at the quantum level

CPT conserved in a local relativistic quantum field theory (vacuum)

$S = \bar{\psi}_1 \psi_2$ scalar

$P = \bar{\psi}_1 \gamma_5 \psi_2$ pseudo scalar

$V_\mu = (V_0, \vec{V})$ $V^\mu = (V_0, -\vec{V})$ vector

$\hookrightarrow \bar{\psi}_1 \gamma_\mu \psi_2$

$A_\mu = \bar{\psi}_1 \gamma_\mu \gamma_5 \psi_2$ axial vector

~~$T_{\mu\nu} = \bar{\psi}_1 \sigma_{\mu\nu} \psi_2$ tensor~~

Parity $V_0 \to V_0$ $\vec{V} \to -V$

$\hat{P}^\dagger \psi(\vec{x}, t) \hat{P} = P_{\alpha\beta} \psi_\beta(-\vec{x}, t)$

$\hat{P}^2 = \hat{I}$ $\hat{P}^\dagger = \hat{P}$ $\to P = \gamma^0$

$S \to S$ $P \to -P$

$V_\mu \to V^\mu$ $A_\mu \to -A^\mu$

Parity

$$\tilde{V}_i = \bar{\psi}_1 \sigma_{0i} \psi_2 \to -\tilde{V}_i \qquad \tilde{A}_i = \epsilon_{ijk} \bar{\psi}_1 \sigma^{jk} \psi_2 \to \tilde{A}_i \qquad i, j, k = 1, 2, 3$$

Charge Conjugation

$$\psi = a^r_{e^-} e^{-ip\cdot x} u_r(p) + (b^+)^r_{e^+} e^{ip\cdot x} v_r(p)$$

annihilate an electron; create a positron

$$\psi_c = b^r_{e^+} e^{-ip\cdot x} u_r(p) + (a^+)^r_{e^-} e^{ip\cdot x} v_r(p)$$

$$e^{i\alpha}(\psi_c)_\alpha = (\hat{C}^+ \psi C)_\alpha = C_{\alpha\beta}\psi^+_\beta$$

$$\hat{C}^2 = \hat{I} \qquad \hat{C}^+ C = 1 \qquad \hat{C} = \hat{C}^+ = \hat{C}^{-1}$$

$$C\gamma^\mu C = -(\gamma^\mu)^*$$

$$C = \pm i\gamma^2$$

$S =$	$\bar{\psi}_1\psi_2$	$\to$	$\bar{\psi}_2\psi_1$
$P =$	$\bar{\psi}_1\gamma_5\psi_2$	$\to$	$\bar{\psi}_2\gamma_5\psi_1$
$V_\mu =$	$\bar{\psi}_1\gamma_\mu\psi_2$	$\to$	$-\bar{\psi}_2\gamma_\mu\psi_1$
$A_\mu =$	$\bar{\psi}_1\gamma_\mu\gamma_5\psi_2$	$\to$	$\bar{\psi}_2\gamma_\mu\gamma_5\psi_1$

$$\bar{\psi}_1\sigma_{\mu\nu}\psi_2 \to -\bar{\psi}_2\sigma_{\mu\nu}\psi_1$$

$$C^+ A_\mu C = -A_\mu$$

photon

$$C^+ W^\pm_\mu C = -W^\mp_\mu$$

charged vector boson

$$e A_\mu \bar{\psi}\gamma^\mu\psi \to$$
$$e(-A_\mu)(-\bar{\psi}\gamma^\mu\psi)$$

invariant

$$g W^+_\mu \bar{d}\gamma^\mu(1-\gamma_5)u$$

P

$$g (W^+)^\mu \bar{d}(\gamma_\mu + \gamma_\mu\gamma_5)u =$$
$$g W^+_\mu \bar{d}\gamma^\mu(1+\gamma_5)u$$

C

$$g(-W^-_\mu)[-\bar{u}\gamma^\mu d - \bar{u}\gamma^\mu\gamma_5 d]$$
$$= g W^-_\mu \bar{u}\gamma^\mu(1+\gamma_5)d$$

NOT DISCUSSED IN THIS LECTURES

4 Inversione temporale T

La meccanica classica è invariante sotto inversione temporale: l'equazione di Newton resta immutata se effettuo la trasformazione $t \longrightarrow -t$.

$$m\frac{dx^2}{dt^2} = V(x)$$

In meccanica quantistica le cose sono più complicate. Riflettiamo anzitutto su cosa è l'inversione temporale. L'evoluzione temporale trasforma certe quantità fisiche $\vec{r}$, $\vec{P}$, E dal tempo 0 al tempo t:

$$\vec{r}(0)\ \vec{P}(0)\ E(0) \longrightarrow \vec{r}(t)\ \vec{P}(t)\ E(t)$$

Definiamo una traslazione in avanti nel tempo:

$$\vec{r}(-T)\ \vec{P}(-T)\ E(-T) \longrightarrow \vec{r}(0)\ \vec{P}(0)\ E(0)$$

Questo è un sistema che evolve dal passato al futuro. Il sistema ottenuto dalla operazione di inversione temporale evolve dal futuro al passato, pertanto:

$$\vec{r}(0)\ \vec{P}(0)\ E(0) \longrightarrow \vec{r}(-T)\ \vec{P}(-T)\ E(-T)$$

Per concretezza si considera il processo $\sigma + \sigma \longrightarrow \pi + \pi$. Il processo inverso temporalmente è $\pi + \pi \longrightarrow \sigma + \sigma$. Se il sistema presenta una simmetria sotto inversione temporale i due processi hanno la stessa ampiezza. Se la simmetria è rotta, i due processi hanno una diversa ampiezza.

4.1 Digressione: antiunitarietà

Sappiamo che l'evoluzione temporale per una funzione d'onda è regolata da:

$$\psi(\mathbf{x}, t) = e^{-iEt}\psi(\mathbf{x}, 0)$$

Applicando la trasformazione $t \longrightarrow -t$ si ottiene:

$$\psi \sim e^{iEt}\psi(\mathbf{x}, 0)$$

Questa espressione è insoddisfacente perché manca il segno $-$ all'esponente. Si osserva però che la ψ^* verifica una elazione soddisfacente:

$$\psi^* \sim e^{-iEt}\psi^*(\mathbf{x}, 0)$$

Consideriamo un generico operatore di evoluzione temporale $\mathbf{U}$ e due stati $|A\rangle$ e $|B\rangle$. Gli stati evoluti temporali saranno:

$$|A'\rangle = \mathbf{U}\,|A\rangle$$
$$|B'\rangle = \mathbf{U}\,|B\rangle$$

Generalmente, per conservare le ampiezze di probabilità, si richiede che:

$$\langle B'|A'\rangle = \langle B|\,\mathbf{U}^{+}\mathbf{U}\,|A\rangle = \langle B|A\rangle$$

ottenendo $\mathbf{U}^{+} = \mathbf{U}$. Nel caso dell'operatore inversione temporale non possiamo procedere in questa maniera. Richiediamo allora che valga:

$$\langle B'|A'\rangle = \langle B|A\rangle^* \tag{12}$$

Un operatore che verifica (12) è detto antiunitario.

Studiamo come si comporta un operatore antiunitario quanto è applicato ad una combinazione lineare di stati. Sia $\langle\psi_j|\psi_i\rangle = \delta_{ij}$ una base ortonormale. Abbiamo:

$$|\phi_i\rangle = \mathbf{U}\,|\psi_i\rangle = \sum_j |\psi_j\rangle\,\langle\psi_j|\,\mathbf{U}\,|\psi_i\rangle = \sum_j U_{ji}\,|\psi_j\rangle$$

dove si è usata la decomposizione dell'identità.

Consideriamo uno stato $|A\rangle$, decomposto in base $|\psi_i\rangle$

$$|A\rangle = \sum_i \langle\psi_i|A\rangle\,|\psi_i\rangle = \sum_i a_i\,|\psi_i\rangle$$

Avremo:

$$|A'\rangle = \mathbf{U}\,|A\rangle = \mathbf{U}\sum_i a_i\,|\psi_i\rangle$$

D'altro canto:

$$|A'\rangle = \sum_i \langle\psi_i'|A'\rangle\,|\psi_i'\rangle = \sum_i \langle\psi_i|A\rangle^*\,|\psi_i'\rangle = \sum_i a_i^*\mathbf{U}\,|\psi_i\rangle$$

Confrontando si conclude che un operatore agisce su una conbinazione lineare di stati restituendo la combinazione lineare delle immagini degli stati, effettuata con i **coefficienti complessi coniugati**.

4.2 Determinazione dell'operatore T

Vogliamo determinare una forma per l'operatore di inversione temporale $\mathbf{T}$. Definiamo l'operatore nel modo seguente:

$$(\psi_T)_\alpha = \left(\mathbf{T}^+\psi(\mathbf{x},t)\mathbf{T}\right)_\alpha = \tau_{\alpha\beta}\psi^*_\beta(\mathbf{x},-t)$$

Quindi:

$$\begin{cases} \psi_T = \mathbf{T}\psi^* \\ \psi_T^+ = (\psi^*)^+\mathbf{T}^+ \end{cases}$$

Se effettuo due volte l'operazione di inversione temporale, il sistema rimane invariato, pertanto $\mathbf{T^2} = \mathbf{I}$.

Vediamo cosa accade alla equazione di Dirac.

$$(i\partial\!\!\!/ - m)\psi = 0$$
$$(i\gamma^0\partial_0 + i\overline{\gamma}\cdot\overline{\partial} - m)\psi = 0$$

Coniugando l'equazione e moltiplicando per $\mathbf{T^2}$ si ottiene:

$$(-i(\gamma^0)^*\partial_0 - i\overline{\gamma}^*\cdot\overline{\partial} - m)\mathbf{T}\mathbf{T}\psi^* = 0$$

Riconoscendo ψ_T e moltiplicando per $\mathbf{T}^+$ da sinistra si ottiene:

$$\mathbf{T}^+(-i(\gamma^0)^*\partial_0 - i\overline{\gamma}^*\cdot\overline{\partial} - m)\mathbf{T}\psi_T = 0$$

Desidero che per la ψ_T valga l'equazione di Dirac seguente:

$$(-i\gamma^0\partial_0 + i\overline{\gamma}\cdot\overline{\partial} - m)\psi_T = 0$$

Scelgo:

$$\mathbf{T} = i\gamma^1\gamma^3$$

Si può verificare che questa scelta soddisfa i requisiti richiesti e $\mathbf{T}^+ = \mathbf{T}$, ovvero l'operatore di inversione temporale è hermitiano.

Let $q(\mathbf{x},t)$ be the Dirac field operator that describes a quark of flavor $q = u, ..., t$, $q^\dagger(\mathbf{x},t)$ denotes its Hermitean adjoint, and $\bar{q} = q^\dagger\gamma^0$. The baryon number operator (36) is

$$\hat{B} = \frac{1}{3}\sum_q \int d^3x : q^\dagger(\mathbf{x},t)q(\mathbf{x},t) : , \tag{109}$$

and the colons denote normal ordering. Let C, P denote the unitary and T the anti-unitary operator which implement the charge conjugation, parity, and time reversal transformations, respectively, in the space of states. Their action on the quark fields is, adopting standard phase conventions,

$$\begin{aligned} Pq(\mathbf{x},t)P^{-1} &= \gamma^0 q(-\mathbf{x},t)\,, & (110)\\ Pq^\dagger(\mathbf{x},t)P^{-1} &= q^\dagger(-\mathbf{x},t)\gamma^0\,, & (111)\\ Cq(\mathbf{x},t)C^{-1} &= i\gamma^2 q^\dagger(\mathbf{x},t)\,, & (112)\\ Cq^\dagger(\mathbf{x},t)C^{-1} &= iq(\mathbf{x},t)\gamma^2\,, & (113)\end{aligned}$$

$$\begin{aligned} Tq(\mathbf{x},t)T^{-1} &= -i\; q(\mathbf{x},-t)\gamma_5\gamma^0\gamma^2\,, & (114)\\ Tq^\dagger(\mathbf{x},t)T^{-1} &= -i\gamma^2\gamma^0\gamma_5 q^\dagger(\mathbf{x},-t)\,, & (115)\end{aligned}$$

where γ^0, γ^2, and $\gamma_5 = i\gamma^0\gamma^1\gamma^2\gamma^3$ denote Dirac matrices. Then

$$\begin{aligned} P : q^\dagger(\mathbf{x},t)q(\mathbf{x},t) : P^{-1} &= : q^\dagger(-\mathbf{x},t)q(-\mathbf{x},t) : , & (116)\\ C : q^\dagger(\mathbf{x},t)q(\mathbf{x},t) : C^{-1} &= : q(\mathbf{x},t)q^\dagger(\mathbf{x},t) := - : q^\dagger(\mathbf{x},t)q(\mathbf{x},t) : , & (117)\\ T : q^\dagger(\mathbf{x},t)q(\mathbf{x},t) : T^{-1} &= : q^\dagger(\mathbf{x},-t)q(\mathbf{x},-t) : \,. & (118)\end{aligned}$$

With these relations we immediately obtain:

$$\begin{aligned} P\hat{B}P^{-1} &= \hat{B}\,, & (119)\\ C\hat{B}C^{-1} &= -\hat{B}\,. & (120)\end{aligned}$$